An Intertemporal CAPM with Higher-Order Moments

Jeewon Jang*

Jangkoo Kang†

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JEL classification: G12

Keywords: Skewness; Intertemporal asset pricing; Risk aversion; Prudence; Anomalies

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* Corresponding author: College of Business, Chosun University, 309 Pilmundaero, Dong-Gu, Gwangju 61452, Republic of Korea; E-mail: jeewonjang@chosun.ac.kr

† College of Business, Korea Advanced Institute of Science and Technology (KAIST), Seoul, Republic of Korea; E-mail: jkkang@business.kaist.ac.kr

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1. Introduction

Since Markowitz (1952) set forth the central theory of portfolio selection, most asset pricing studies have been developed based on only the first and second moments of a return distribution. The Capital Asset Pricing Model (CAPM) of Sharpe (1964) and Lintner (1965) is an exemplar of mean–variance models, and it still serves as a theoretical benchmark to other subsequent asset pricing models, despite its empirical shortcomings. Most of the succeeding research has endeavored to relax the restrictive conditions involved in deriving the CAPM. Among other studies, by generalizing the simple one-period setting to a multi-period one, Merton (1973) and Long (1974) extend the static CAPM to the intertemporal versions, respectively in continuous-time and discrete-time frameworks.

Mean–variance portfolio analysis, to a great extent, relies on one of two critical assumptions: the quadratic utility of a representative agent in the economy, or the normally distributed returns of risky assets. If neither assumption can be satisfied, the mean–variance portfolio theory is valid only approximately, not exactly. Unfortunately, there is much evidence to suggest that both assumptions are violated. Although the quadratic utility is the simplest functional form to describe investor risk aversion, it is not appealing theoretically, since it cannot satisfy the assumption of decreasing absolute risk aversion.[1] Further, there is multiple documentation that stock returns are not normally distributed; they are known to be both skewed and fat-tailed. More specifically, one stylized fact is that the distribution of aggregate stock returns exhibits negative skewness, while individual stock returns are often positively skewed (Albuquerque, 2012). Another stylized fact is that stock returns also display significant kurtosis (see Mandelbrot, 1963; Fama, 1965). All the evidence implies that the optimal portfolio selection cannot be characterized completely by mean and variance alone.

Given the weakness of the mean–variance portfolio theory, the question arises of whether moments of higher order than the variance, in particular skewness and kurtosis of a return distribution, affect expected rates of return on risky assets as well. If it could be assumed that

[1] Decreasing absolute risk aversion is a widely accepted assumption, since it implies seemingly reasonable behavior such that one invests more in risky assets as one becomes wealthier.

risk-averse investors consistently prefer higher skewness and lower kurtosis, the observation that aggregate stock returns are negatively skewed and leptokurtic may be a source of systematic underestimation of the equity premium by the existing mean–variance models. To address these concerns, there has been a strand of literature that takes higher-order moments into account in equilibrium expected rates of return. Kraus and Litzenberger (1976) put forth the three-moment CAPM, incorporating the effect of skewness on valuation of risky assets, based on preference for positive skewness. The model shows that systematic skewness, rather than total skewness, is relevant to asset pricing, suggesting that the poor empirical performance of the traditional CAPM may be attributable to omission of systematic skewness. Though kurtosis and its effect on expected returns have attracted relatively little attention, the three-moment CAPM can be extended to include any number of higher-order moments without difficulty.[2] Following their seminal work, more recent studies provide empirical evidence that higher-order moments of a return distribution play a significant role in pricing stocks. Harvey and Siddique (2000) and Dittmar (2002) show that an individual asset's co-moments with the aggregate market portfolio should be priced if the pricing kernel is assumed as a nonlinear function of the market return. Focusing on the third and the fourth moments, respectively, they also document empirical evidence that co-moments are indeed priced in the U.S. stock market.

Although pricing of co-skewness and co-kurtosis is supported by both theoretical argument and empirical evidence, there appears to be little consensus about the use of asset pricing models with higher-order moments. This may be ascribed to two main weaknesses of the existing literature on co-skewness and co-kurtosis. One serious drawback is that none of the existing models explore intertemporal considerations for the pricing of higher-order moments in expected stock returns; they take only a static setting into account. However, the static asset pricing models are generally not consistent with investor optimization, unless the investment opportunities do not change over time, or at least vary in deterministic ways. With

[2] Though Kraus and Litzenberger (1976) argue that there is no reason to consider the fourth or higher-order moments in the model, kurtosis, unlike other moments of even higher order, is well worth considering, in that stock returns are observed to be highly leptokurtic. Kurtosis is known as a measure of heavy tails in a probability distribution, while it is difficult to deal with moments of higher order than kurtosis, since what they mean about the shape of a distribution is not clear.

a more realistic assumption of stochastic investment opportunities, asset pricing models could properly reflect long-term investors' hedging demand against deterioration of investment opportunities in the valuation equation. The failure of the existing models to capture intertemporal hedging demands may undermine the importance of pricing of co-skewness and co-kurtosis, despite their theoretical basis.

The other drawback of the existing literature is that it does not adequately address measurement errors in the estimation procedure. Most empirical asset pricing studies estimate their models using two-pass cross-sectional regressions, in which co-moments of higher order as well as betas are estimated in the first stage and then the estimated co-moments are used subsequently in the second-stage estimation. Since higher-order moments are more difficult to measure precisely than variance, larger measurement errors occur in the first-stage estimation for higher moment models than for mean–variance models. More importantly, larger errors in the first stage, combined with the fact that higher moment models include a larger number of estimates, can lead to huge estimation errors for the price of risk in the second stage. Even though the errors-in-variables problem should be handled more carefully for higher moment models to obtain consistent estimates in the second stage, existing empirical studies with co-moments of higher order have not paid close attention to this matter. This may lead to biases in the cross-sectional estimation results that are possibly much larger than in the mean–variance models.

This paper extends the literature by incorporating the effects of both higher-order moments and stochastic investment opportunities on expected stock returns in a theoretical model, and by overcoming the difficulty related to the errors-in-variables problem in estimating the model. Specifically, we present an intertemporal asset pricing model with the assumption that returns of risky assets could have any distribution with non-zero and finite skewness and kurtosis. The resulting pricing equation encompasses the three- and four-moment static CAPM, as well as the traditional CAPM. In addition, our model encompasses the intertemporal CAPM based on mean–variance optimization. We also estimate our asset pricing model based on a generalized two-stage procedure that can handle the errors-in-variables problem in the higher-order moment model. The estimation results using U.S. stock market data produce empirical evidence supporting our theoretical model. We further explore implications of the model on various asset pricing anomalies unexplained by a mean–variance trade-off.

Our theoretical findings are summarized as follows. We propose a four-moment intertemporal asset pricing model in a discrete-time framework with the assumptions that (i) a representative investor has preference for high skewness and low kurtosis, (ii) the investment opportunities available to investors are stochastic, and (iii) the joint probability distribution of future wealth and state variables has finite fourth- and lower-order moments. We find that, in equilibrium, the expected rate of return on a risky asset is proportional to the asset's covariance, co-skewness, and co-kurtosis with market returns, consistent with the results of Harvey and Siddique (2000) and Dittmar (2002). Further, the expected return on a risky asset should include extra premiums for the asset's covariance with each state variable, co-skewness with the market returns and each state variable, and co-kurtosis with the squared market returns and each state variable, due to the stochastic investment opportunities. The return premiums for co-skewness and co-kurtosis related with state variables indicate that investors require compensation for an increase in systematic risk and a decrease in systematic skewness associated with deterioration of investment opportunities.

We document empirical evidence supporting the predictions of the four-moment intertemporal CAPM using U.S. stock market data. Specifically, using daily returns of individual stocks during the period 1926 to 2012 and various state variables as a proxy for future investment opportunities, we investigate whether higher-order co-moments are indeed priced in the cross-section. Via both portfolio sorts and cross-sectional regressions, we find strong and robust evidence that systematic skewness is significantly priced, and that the price of systematic skewness is negative. This is consistent with theoretical predictions based on skewness preference, as well as previous empirical results in the literature. Moreover, we find that the third-order co-moment with the market return and each state variable has a significant and negative price when the state variable predicts good states, as in the case of the term spread and the *HML* factor. This can be interpreted as that such state variables are related to variations in risk aversion, and that higher return is required on a stock with lower co-moment as compensation for an increase in systematic risk associated with a rise in risk aversion. However, premiums for systematic kurtosis and most of the fourth-order co-moments become insignificant after inclusion of systematic variance and skewness.

We also explore time-series and cross-sectional implications of the four-moment ICAPM for asset pricing anomalies based on the estimation results. First, we examine relative risk aversion (RRA) and relative prudence (RPR) implied by the estimated models, and confirm that there is an intertemporal relation of expected returns, risk, and skewness. That is, at a given level of variance, there is a trade-off between expected returns and negative skewness, whereas there is a trade-off between returns and risk at a given level of skewness. Second, we investigate whether dispersion in higher-order co-moments, not only dispersion in systematic risk, is related to cross-sectional patterns in stock returns for several well-known anomalies: the size and value effect, price momentum, idiosyncratic volatility puzzle, and financial distress puzzle. The results suggest that, at least partially, the value premium can be a reward for bearing low systematic skewness, that the high return on a low volatility stock can be due to its cyclical variations, and that the abnormal returns from strategies based on momentum and failure probability come from compensation for accepting higher systematic risk associated with high risk aversion.

Overall, our findings indicate that stock market investors consistently prefer positive skewness, and that there exists a significant risk-return-skewness relation, whereas there is no clear evidence on kurtosis preference. In addition, our empirical findings show that risk-averse investors demand a hedge against intertemporal changes in systematic risk, indicating the importance of intertemporal considerations in asset pricing theories. These suggest that the failure of the existing asset pricing theories in explaining several anomalies is possibly due to omission of systematic skewness and third-order co-moments related to intertemporal hedging demands.

Our contribution to the literature is twofold. First, we provide a theory based on intertemporal optimization of an investor with preference for higher-order moments; this has not been explored in previous studies, and it encompasses a wide range of existing theories. We add to them by identifying extra premiums required for an increase in systematic risk and a decrease in systematic skewness associated with deterioration of investment opportunities. On the other hand, we also provide a strong proof for them by confirming that their predictions are not artifacts derived from restrictive, unrealistic assumptions. Second, our empirical findings highlight the role of systematic skewness and the third-order co-moments related to intertemporal hedging demands in future research. We produce strong evidence for an intertemporal relation of expected returns, risk, and skewness, which is not discussed in the literature, based on results estimated precisely from the generalized procedure. This suggests that third-order moments may play a critical role in resolving asset pricing anomalies unexplained by a risk-return trade-off alone. We expect that future empirical research on the role of higher-order moments will advance our understanding of asset pricing.

The remainder of this paper proceeds as follows. Section 2 presents a four-moment intertemporal asset pricing model and derives the equation for equilibrium expected returns on risky assets. Section 3 presents the empirical framework and provides estimation results from cross-sectional regressions. Section 4 discusses time-series and cross-sectional implications of the model and explores the role of higher-order moments in asset pricing anomalies. Section 5 concludes the paper.

2. Theoretical Model

We derive a four-moment intertemporal asset pricing model that incorporates the effects of both higher-order moments and stochastic investment opportunities on the valuation of risky assets. The resulting model encompasses the static three- and four-moment CAPM in Kraus and Litzenberger (1976), Harvey and Siddique (2000), and Dittmar (2002). We also present an economic interpretation of prices of risk by relating them to investor risk attitudes.

2.1 Derivation of a Four-Moment Intertemporal CAPM

There are N risky assets and a risk-free asset in the economy, traded at times spaced discretely. A representative consumer-investor chooses an optimal investment in risky assets to maximize expected utility. To focus on the optimal investment decision, it is assumed that there is only one good for consumption in the economy and that the remaining amount of wealth not invested is used for current consumption. The amount of consumption at time t is denoted by C_t in nominal dollar value, and the amounts of investment in the risk-free asset and in the N risky assets at time t are denoted by x_t and $y_t = (y_{1t}, \ldots, y_{Nt})'$, respectively, also in dollar values. Then, the nominal wealth level of the investor at time t, W_t, is represented as $W_t = C_t + x_t + y_t'\mathbf{1}_N$.

The utility function of the investor is not restricted to a specific form, but is assumed to be four times differentiable and time-separable. We follow the standard assumptions of positive marginal utility and risk aversion, that is, $u' > 0$ and $u'' < 0$. Additionally, to determine the preference direction for skewness and kurtosis, we further assume that the investor is prudent and temperate, i.e., $u''' > 0$ and $u'''' < 0$.[3] As Fama (1970) has pointed out, the multi-period consumption-investment decision can always be treated as the one-period problem in which the agent at time 0 is maximizing the expected value of an indirect utility function at time 1. To take uncertainty in future investment opportunities into account, we assume that any changes in the investment opportunity set at time t are summarized by K state variables, denoted as $z_t = (z_{1t}, \ldots, z_{Kt})'$. Then, the indirect utility function at time t can be represented as a function of the investor's wealth and K state variables at time t, denoted as $V_t(W_t, z_t)$.

Now the investor's intertemporal consumption-investment decision problem at time 0 can be stated as follows:

$$\max_{\{C_0, x_0, y_0\}} u(C_0) + \beta E_0\left[V_1(W_1, z_1)\right] \ \text{ s.t } \ C_0 + x_0 + y_0'\mathbf{1}_N = W_0 \ , \quad x_0 \cdot R_{f0} + y_0' R_1 = W_1 \ , \qquad (1)$$

where R_{ft} and $R_t = (R_{1t}, \ldots, R_{Nt})$ are gross returns of the risk-free and the N risky assets, respectively, β is the discount parameter, and $E_t[\cdot]$ indicates the expectation operator conditional on the information set available to the investor at time t.

From the above statement, we can determine that the investor's decision depends on the joint conditional probability distribution of (W_1, z_1). The simplest assumption is that (W_1, z_1) is distributed with the multivariate normal, as in the literature based on mean–variance portfolio analysis, including Long (1974). The normality assumption simplifies the problem, because the determinants of the expected utility can be summarized with only the first and

[3] Scott and Horvath (1980) show that, for the usual investor who is consistent in the direction of preference for moments, the preference direction is positive for every odd central moment and negative for every even central moment.

second moment parameters of the multivariate normal distribution. We do not follow this convention, however, to take moments of higher order than the variance into consideration. Further, we assume no specific probability distribution for (W_1, z_1), allowing any multivariate distribution with finite fourth and lower-order moments.

Accordingly, the expected utility of the investor is assumed to be determined by the fourth and lower-order moments of the joint probability distribution of (W_1, z_1), that is, its mean, variance, skewness, and kurtosis. Note that these moments of the multivariate distribution are defined as matrices with co-moments among each variable, which are defined precisely shortly.[4] Also note that, when considering co-moments between variables, only relevant ones in the consumption-investment decision problem are co-moments with wealth, W_1, which means that co-moments among K state variables themselves would not affect the expected utility.

Based on the preceding distributional assumptions, the investor's problem given in equation (1) can be restated using the relevant moments of the joint probability distribution (conditional on the information set at time t). Definitions for all relevant moments are given as follows:

$$m_t = E_t(W_{t+1}) = (W_t - C_t - y_t'\mathbf{1}_N)R_{ft} + y_t'\mu_t = (W_t - C_t)R_{ft} + y_t'(\mu_t - \mathbf{1}_N R_{ft}),$$

where $\mu_t = E_t(R_{t+1})$ is the (N x 1) vector of mean returns of the N risky assets,

$$v_t = Var_t(W_{t+1}) = y_t'\Sigma_t y_t,$$

where $\Sigma_t = Var_t(R_{t+1})$ is the (N x N) matrix of covariance among the N risky assets,

$$h_t = Cov_t(W_{t+1}, z_{t+1}) = y_t'\Phi_t \qquad (1 \text{ x } K),$$

where $\Phi_t = Cov_t(R_{t+1}, z_{t+1})$ is the (N x K) matrix of covariance between the N risky assets and the K state variables,

$$s_t = Skew_t(W_{t+1}) = E_t\left[(W_{t+1} - m_t)^3\right] = E_t\left[\{y_t'(R_{t+1} - \mu_t)\}^3\right] = y_t'\Theta_t(y_t \otimes y_t),$$

---

[4] Though there may be many ways to define the multivariate skewness and kurtosis, we follow the definition of the so-called higher-order moment tensors introduced in Harvey et al. (2010), Jondeau and Rockinger (2006), and Martellini and Ziemann (2010).

where $\Theta_t = E_t\left[(R_{t+1}-\mu_t)(R_{t+1}-\mu_t)' \otimes (R_{t+1}-\mu_t)'\right]$ is the (N x N^2) matrix of co-skewness among the N risky assets,

$$\begin{aligned} l_t &= Coskew_t(W_{t+1}, z_{t+1}) = E_t\left[(W_{t+1}-m_t)(W_{t+1}-m_t)' \otimes (z_{t+1}-E_t(z_{t+1}))'\right] \\ &= E_t\left[y_t'(R_{t+1}-\mu_t)(R_{t+1}-\mu_t)' y_t \otimes (z_{t+1}-E_t(z_{t+1}))'\right] = y_t'\Psi_t(y_t \otimes I_K) \qquad (1 \text{ x } K), \end{aligned}$$

where $\Psi_t = E_t\left[(R_{t+1}-\mu_t)(R_{t+1}-\mu_t)' \otimes (z_{t+1}-E_t(z_{t+1}))'\right]$ is the (N x NK) matrix of co-skewness between the N risky assets and the K state variables,

$$k_t = Kurt_t(W_{t+1}) = E_t\left[(W_{t+1}-m_t)^4\right] = E_t\left[\{y_t'(R_{t+1}-\mu_t)\}^4\right] = y_t'\Pi_t(y_t \otimes y_t \otimes y_t),$$

where $\Pi_t = E_t\left[(R_{t+1}-\mu_t)(R_{t+1}-\mu_t)' \otimes (R_{t+1}-\mu_t)' \otimes (R_{t+1}-\mu_t)'\right]$ is the (N x N^3) matrix of co-kurtosis among the N risky assets,

$$\begin{aligned} n_t &= Cokurt_t(W_{t+1}, z_{t+1}) = E_t\left[(W_{t+1}-m_t)(W_{t+1}-m_t)' \otimes (W_{t+1}-m_t)' \otimes (z_{t+1}-E_t(z_{t+1}))'\right] \\ &= E_t\left[y_t'(R_{t+1}-\mu_t)(R_{t+1}-\mu_t)' y_t \otimes (R_{t+1}-\mu_t)' y_t \otimes (z_{t+1}-E_t(z_{t+1}))'\right] \\ &= y_t'\Omega_t(y_t \otimes y_t \otimes I_K) \qquad (1 \times K), \end{aligned}$$

where $\Omega_t = E_t\left[(R_{t+1}-\mu_t)(R_{t+1}-\mu_t)' \otimes (R_{t+1}-\mu_t)' \otimes (z_{t+1}-E_t(z_{t+1}))'\right]$ is the (N x N^2K) matrix of co-kurtosis between N risky assets and K state variables.

Given the joint conditional probability distribution of (W_{t+1}, z_{t+1}), there exists a function G at any time t that satisfies the following property:

$$G(C_t, m_t, v_t, h_t, s_t, l_t, k_t, n_t) = u(C_t) + \beta E_t\left[V_{t+1}(W_{t+1}, z_{t+1})\right] . \qquad (2)$$

Now, the problem in equation (1) can be restated using the moments of the joint conditional probability distribution of (W_1, z_1):

$$\max_{\{C_0, y_0\}} G(C_0, m_0, v_0, h_0, s_0, l_0, k_0, n_0) . \qquad (3)$$

The first-order condition with respect to y_0 to get the optimal amount of risky investments is obtained as in the following:

$$
\begin{aligned}
0 = & \frac{\partial G}{\partial m_0} \cdot (\mu_0 - \mathbf{1}_N R_{f0}) + \frac{\partial G}{\partial v_0} \cdot 2\Sigma_0 y_0 + \Phi_0 \frac{\partial G}{\partial h_0'} \\
& + \frac{\partial G}{\partial s_0} \cdot 3\Theta_0 (y_0 \otimes y_0) + 2\Psi_0 (y_0 \otimes I_K) \cdot \frac{\partial G}{\partial l_0'} \\
& + \frac{\partial G}{\partial k_0} \cdot 4\Pi_0 (y_0 \otimes y_0 \otimes y_0) + 3\Omega_0 (y_0 \otimes y_0 \otimes I_K) \cdot \frac{\partial G}{\partial n_0'} \ .
\end{aligned} \tag{4}
$$

Since the first-order condition in equation (4) is not linear in y_0, the closed-form solution for the optimal allocation of risky assets cannot be calculated. Nevertheless, the expected returns of risky assets that clear the market can be obtained with additional assumptions on the securities market. We assume that there are I identical investors in the market, and denote the total amount of market supply of the N risky assets in dollars as $V = (V_1, \ldots, V_N)$. These two additional assumptions indicate that the market clearing condition should be $I \cdot y_0 = V$.

Investors' first-order condition in equation (4) and the market clearing condition jointly imply that the following equation for the expected returns of risky assets should hold in market equilibrium:

$$
\begin{aligned}
0 = & I \cdot G_m \cdot (\mu_0 - \mathbf{1}_N R_{f0}) + G_v \cdot 2\Sigma_0 V + I \cdot \Phi_0 \cdot G_h' \\
& + \frac{1}{I} \cdot G_s \cdot 3\Theta_0 (V \otimes V) + 2\Psi_0 (V \otimes I_K) \cdot G_l' \\
& + \frac{1}{I^2} \cdot G_k \cdot 4\Pi_0 (V \otimes V \otimes V) + \frac{1}{I} \cdot 3\Omega_0 (V \otimes V \otimes I_K) \cdot G_n' \ ,
\end{aligned} \tag{5}
$$

where G_x means the partial derivative of the function G with respect to a variable x.

The equilibrium rates of return on the N risky assets, implied by equation (5), are presented in the following equation (for the mathematical derivation, see Appendix A):

$$
\begin{aligned}
E_t\left[R_{t+1}-R_{ft}\right] = & -\frac{2(\mathbf{1}_N' V)}{I}\cdot\frac{G_v}{G_m}\cdot E_t\left[(R_{t+1}-\mu_t)(R_{t+1}^M-\mu_t^M)\right] \\
& -E_t\left[(R_{t+1}-\mu_t)(z_{t+1}-E_t(z_{t+1}))'\right]\cdot\frac{G_h'}{G_m} \\
& -\frac{3(\mathbf{1}_N' V)^2}{I^2}\cdot\frac{G_s}{G_m}\cdot E_t\left[(R_{t+1}-\mu_t)(R_{t+1}^M-\mu_t^M)^2\right] \\
& -\frac{2(\mathbf{1}_N' V)}{I}\cdot E_t\left[(R_{t+1}-\mu_t)(R_{t+1}^M-\mu_t^M)(z_{t+1}-E_t(z_{t+1}))'\right]\cdot\frac{G_l'}{G_m} \\
& -\frac{4(\mathbf{1}_N' V)^3}{I^3}\cdot\frac{G_k}{G_m}\cdot E_t\left[(R_{t+1}-\mu_t)(R_{t+1}^M-\mu_t^M)^3\right] \\
& -\frac{3(\mathbf{1}_N' V)^2}{I^2}\cdot E_t\left[(R_{t+1}-\mu_t)(R_{t+1}^M-\mu_t^M)^2(z_{t+1}-E_t(z_{t+1}))'\right]\cdot\frac{G_n'}{G_m}\ , \qquad (6)
\end{aligned}
$$

where $R_{t+1}^M = (\mathbf{1}_N' V)^{-1} V' R_{t+1}$ indicates gross return of the value-weighted portfolio of the N risky assets, and $\mu_t^M = E_t(R_{t+1}^M) = (\mathbf{1}_N' V)^{-1} V' \mu_t$ indicates its mean.

The resulting equation shows that the expected return on a risky asset is determined proportionally by (i) its covariance with returns on the market portfolio, (ii) its co-skewness with market returns, and (iii) its co-kurtosis with market returns, regardless of whether the investment opportunities are stochastic. In addition, when the investment opportunities are stochastic, the expected return on a risky asset should include extra premiums for (iv) its covariance with each state variable, (v) its co-skewness with market returns and each state variable, and (vi) its co-kurtosis with squared market returns and each state variable. Note that the equation is reduced to the static four-moment CAPM if the investment opportunities are assumed to be deterministic. The terms of co-moments with state variables are additionally introduced in our model, to capture intertemporal hedges against deterioration of investment opportunities.

2.2 Interpretation with Mean-Variance-Skewness-Kurtosis Approximation

To present an economic interpretation on price of each co-moment term in the pricing equation, we suppose in this subsection that the expected utility of the representative investor can be approximated as a function of mean, variance, skewness, and kurtosis of the investor's wealth and state variables. The four-moment approximation leads to interpretation on our pricing equation in more intuitive way, which is consistent with investor preference.

To this end, we consider the investor's utility as a function of wealth and K state variables. Further, it is assumed that the utility becomes zero when differentiated with respect to any state variables more than twice (i.e., u_{zz} = 0, u_{wzz} = 0, u_{zzz} = 0, u_{wwzz} = 0, u_{wzzz} = 0, u_{zzzz} = 0). The fourth-order Taylor expansion of the utility function at time 1 around the conditional mean of (W_1, z_1) yields that:

$$\begin{aligned} u(W_1, z_1) \simeq\ & u(m_0, \overline{z}_0) + u_w(m_0, \overline{z}_0)(W_1 - m_0) + (z_1 - \overline{z}_0)' u_z(m_0, \overline{z}_0) \\ & + \frac{1}{2}\left[u_{ww}(m_0, \overline{z}_0)(W_1 - m_0)^2 + 2(W_1 - m_0)(z_1 - \overline{z}_0)' u_{wz}(m_0, \overline{z}_0)\right] \\ & + \frac{1}{6}\left[u_{www}(m_0, \overline{z}_0)(W_1 - m_0)^3 + 3(W_1 - m_0)^2 (z_1 - \overline{z}_0)' u_{wwz}(m_0, \overline{z}_0)\right] \\ & + \frac{1}{24}\left[u_{wwww}(m_0, \overline{z}_0)(W_1 - m_0)^4 + 4(W_1 - m_0)^3 (z_1 - \overline{z}_0)' u_{wwwz}(m_0, \overline{z}_0)\right], \end{aligned} \tag{7}$$

where $\overline{z}_0$ denotes the conditional mean of z_1 at time 0. By taking the conditional expectation on both sides of (7) and using the notation for the conditional moments defined above, the expected utility can be approximated as a linear function of the moments that could affect it.

$$\begin{aligned} G &= E_0\left[u(W_1, z_1)\right] \\ &\simeq u(m_0, \overline{z}_0) + \frac{1}{2} u_{ww}(m_0, \overline{z}_0) \cdot v_0 + h_0 \cdot u_{wz}(m_0, \overline{z}_0) + \frac{1}{6} u_{www}(m_0, \overline{z}_0) \cdot s_0 \\ &\quad + \frac{1}{2} l_0 \cdot u_{wwz}(m_0, \overline{z}_0) + \frac{1}{24} u_{wwww}(m_0, \overline{z}_0) \cdot k_0 + \frac{1}{6} n_0 \cdot u_{wwwz}(m_0, \overline{z}_0) \end{aligned} \tag{8}$$

Now we define a new variable, $i_t = \dfrac{\mathbf{1}_N' V}{m_t I} = \dfrac{1}{\overline{W}} \cdot \dfrac{\mathbf{1}_N' V}{I} = \dfrac{\mathbf{1}_N' y_t}{\overline{W}}$, which represents the ratio of the amount of risky investments to the mean wealth level, or the investment–wealth ratio.[5] From this new definition and the approximated formula in (8), we can express the price of

[5] If we let w denote the proportion of risky investments in $W_t - C_t$, the investment-to-wealth ratio, i_t, can be represented as $w \big/ \left\{(1-w)R_{ft} + w\mu_t^M\right\}$. This indicates the ratio of risky investments to the expected payoff of total investments in the next period.

each co-moment in the equilibrium expected returns of (6) in terms of parameters indicating investor preference and risk attitude, as follows:

$$-\frac{2(\mathbf{1}_N' V)}{I}\cdot\frac{G_v}{G_m} = -2i_t m_t \cdot \frac{1}{2}\cdot\frac{u_{ww}(m_t,\overline{z}_t)}{u_w(m_t,\overline{z}_t)} = i_t \cdot \left(\frac{-u_{ww}(m_t,\overline{z}_t)m_t}{u_w(m_t,\overline{z}_t)}\right) = i_t \cdot \gamma_t \,,$$

where γ_t is relative risk aversion at the mean level of wealth,

$$-\frac{G_h'}{G_m} = -\frac{u_{wz}(m_t,\overline{z}_t)}{u_w(m_t,\overline{z}_t)} = -\delta_t \,,$$

where $\delta_t = \frac{u_{wz}(m_t,\overline{z}_t)}{u_w(m_t,\overline{z}_t)}$ is a (K x 1) vector that measures how much the marginal utility increases as each of the state variables increases,

$$-\frac{3(\mathbf{1}_N' V)^2}{I^2}\cdot\frac{G_s}{G_m} = -3i_t^2 m_t^2 \cdot \frac{1}{6}\cdot\frac{u_{www}(m_t,\overline{z}_t)}{u_w(m_t,\overline{z}_t)} = -\frac{i_t^2}{2}\cdot\left(\frac{-u_{ww}(m_t,\overline{z}_t)m_t}{u_w(m_t,\overline{z}_t)}\right)\cdot\left(\frac{-u_{www}(m_t,\overline{z}_t)m_t}{u_{ww}(m_t,\overline{z}_t)}\right)$$
$$= -\frac{i_t^2}{2}\cdot\gamma_t\cdot\theta_t \;,$$

where θ_t is a measure of relative prudence at the mean level of wealth,

$$-\frac{2(\mathbf{1}_N' V)}{I}\cdot\frac{G_l'}{G_m} = -2i_t m_t \cdot \frac{1}{2}\cdot\frac{u_{wwz}(m_t,\overline{z}_t)}{u_w(m_t,\overline{z}_t)} = i_t \cdot \left(\frac{-u_{ww}(m_t,\overline{z}_t)m_t}{u_w(m_t,\overline{z}_t)}\right)\cdot\left(\frac{u_{wwz}(m_t,\overline{z}_t)}{u_{ww}(m_t,\overline{z}_t)}\right)$$
$$= i_t\cdot\gamma_t\cdot\lambda_t \;,$$

where $\lambda_t = \frac{u_{wwz}(m_t,\overline{z}_t)}{u_{ww}(m_t,\overline{z}_t)}$ is a (K x 1) vector that measures how much risk aversion increases as each of the state variables increases,

$$-\frac{4(\mathbf{1}_N' V)^3}{I^3}\cdot\frac{G_k}{G_m} = -4i_t^3 m_t^3 \cdot \frac{1}{24}\cdot\frac{u_{wwww}(m_t,\overline{z}_t)}{u_w(m_t,\overline{z}_t)}$$
$$= \frac{i_t^3}{6}\cdot\left(\frac{-u_{ww}(m_t,\overline{z}_t)m_t}{u_w(m_t,\overline{z}_t)}\right)\cdot\left(\frac{-u_{www}(m_t,\overline{z}_t)m_t}{u_{ww}(m_t,\overline{z}_t)}\right)\cdot\left(\frac{-u_{wwww}(m_t,\overline{z}_t)m_t}{u_{www}(m_t,\overline{z}_t)}\right)$$
$$= \frac{i_t^3}{6}\cdot\gamma_t\cdot\theta_t\cdot\eta_t \;,$$

where η_t is a measure of relative temperance at the mean level of wealth,[6]

$$
\begin{aligned}
-\frac{3(\mathbf{1}'_N V)^2}{I^2}\cdot\frac{G'_n}{G_m} &= -3i_t^2 m_t^2\cdot\frac{1}{6}\cdot\frac{u_{wwwz}(m_t,\overline{z}_t)}{u_w(m_t,\overline{z}_t)} \\
&= -\frac{i_t^2}{2}\cdot\left(\frac{-u_{ww}(m_t,\overline{z}_t)m_t}{u_w(m_t,\overline{z}_t)}\right)\cdot\left(\frac{-u_{www}(m_t,\overline{z}_t)m_t}{u_{ww}(m_t,\overline{z}_t)}\right)\cdot\left(\frac{u_{wwwz}(m_t,\overline{z}_t)}{u_{www}(m_t,\overline{z}_t)}\right) \\
&= -\frac{i_t^2}{2}\cdot\gamma_t\cdot\theta_t\cdot\rho_t\ ,
\end{aligned}
$$

where $\rho_t = \frac{u_{wwwz}(m_t,\overline{z}_t)}{u_{www}(m_t,\overline{z}_t)}$ is a (K x 1) vector that measures how much prudence increases as each of the state variables increases.

From the four-moment approximation of the expected utility, the asset pricing equation in (6) can be rewritten as follows:

$$
\begin{aligned}
E_t\left[R_{t+1}-R_{ft}\right] = {} & i_t\cdot\gamma_t\cdot E_t\left[(R_{t+1}-\mu_t)(R_{t+1}^M-\mu_t^M)\right] - E_t\left[(R_{t+1}-\mu_t)(z_{t+1}-E_t(z_{t+1}))'\right]\cdot\delta_t \\
& -\frac{i_t^2}{2}\cdot\gamma_t\cdot\theta_t\cdot E_t\left[(R_{t+1}-\mu_t)(R_{t+1}^M-\mu_t^M)^2\right] \\
& +i_t\cdot\gamma_t\cdot E_t\left[(R_{t+1}-\mu_t)(R_{t+1}^M-\mu_t^M)(z_{t+1}-E_t(z_{t+1}))'\right]\cdot\lambda_t \\
& +\frac{i_t^3}{6}\cdot\gamma_t\cdot\theta_t\cdot\eta_t\cdot E_t\left[(R_{t+1}-\mu_t)(R_{t+1}^M-\mu_t^M)^3\right] \\
& -\frac{i_t^2}{2}\cdot\gamma_t\cdot\theta_t\cdot E_t\left[(R_{t+1}-\mu_t)(R_{t+1}^M-\mu_t^M)^2(z_{t+1}-E_t(z_{t+1}))'\right]\cdot\rho_t\ . \qquad (9)
\end{aligned}
$$

An interpretation of the equation of equilibrium expected returns in (9) is given as follows. Risk-averse investors require a higher rate of return on a risky asset whose covariance with market return is higher—the more risk-averse investors are, the higher the return premium required. Prudent investors require a higher rate of return on a risky asset whose co-skewness with market return is lower—the more prudent investors are, the higher the return premium required. Temperate investors require a higher rate of return on a risky asset whose co-

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[6] Kimball (1992) introduces the concept of temperance to refer that an unavoidable background risk should lead an investor to behave in a more risk-averse way for another risk, even if the two risks are statistically independent. The risk-averse and prudent investor is temperate if and only if the fourth derivative of his utility is negative, i.e., $u'''' < 0$.

kurtosis with market return is higher—the more temperate investors are, the higher the return premium required. If investment opportunities change over time stochastically, expected return is determined in proportion to the covariance between asset return and state variables. For a state variable that causes the marginal utility to increase as it increases (i.e., $\delta_t > 0$), the higher rate of return is required as the covariance is lower. Expected return is determined in proportion to the co-skewness among asset return, market return, and state variables. For a state variable that causes risk aversion to increase as it increases (i.e., $\lambda_t > 0$), the higher rate of return is required as the co-skewness is higher, since high co-skewness indicates an increase in systematic risk in an unfavorable state. Expected return is determined in proportion to the co-kurtosis among asset return, squared market return, and state variables. For a state variable that causes prudence to increase as it increases (i.e., $\rho_t > 0$), the higher rate of return is required as the co-kurtosis is lower, since low co-kurtosis indicates a decrease in systematic skewness in an unfavorable state.

Our model encompasses the existing asset pricing models. The model is reduced to (i) the static CAPM of Sharpe (1964) and Lintner (1965) if it is assumed that the investment opportunities are constant, and that prudence and temperance are zero (i.e., $\theta_t = \eta_t = 0$) or the asset returns are normally distributed; (ii) the three-moment static CAPM of Kraus and Litzenberger (1976) and Harvey and Siddique (2000) if it is assumed that the investment opportunities are constant, and that temperance is zero (i.e., $\eta_t = 0$) or the asset returns have zero kurtosis; (iii) the four-moment static CAPM of Dittmar (2002) if it is assumed that the investment opportunities are constant; (iv) the intertemporal CAPM of Merton (1973) and Long (1974) if it is assumed that prudence and temperance are zero (i.e., $\theta_t = \eta_t = 0$) or the asset returns are normally distributed, and that risk aversion does not change with the investment opportunity (i.e., $\lambda_t = 0$).

In addition to the existing models, our model further captures compensation for co-skewness and co-kurtosis among asset return, market return, and state variables. Premiums for these third- and fourth-order co-moments indicate intertemporal hedging demands of the long-term investor who has preference for low variance, high skewness, and low kurtosis, for unfavorable changes in systematic risk and systematic skewness, respectively.

3. Empirical Results

In this section, we present empirical evidence for the four-moment ICAPM using U.S. stock market data. Employing various state variables as predictors of future investment opportunities, we investigate whether co-moments in the asset pricing equation derived theoretically are priced in the cross-section of stock returns. We also estimate prices of co-moments for various model specifications, using two-pass cross-sectional regressions.

3.1 Data and Methodology

For the empirical investigation, we use daily returns of individual stocks from the CRSP database. Specifically, our sample includes all NYSE/AMEX/NASDAQ ordinary common stocks with CRSP share codes 10 and 11 over the period 1926 to 2012. In each year during the period, stocks with less than 126 observations of daily returns are excluded. We use the CRSP value-weighted index as the market portfolio and the one-month Treasury bill rate as the risk-free rate.

To explore the role of stochastic investment opportunities in the four-moment ICAPM empirically, it is essential to choose appropriate state variables to predict a change in the future investment opportunity set. Based on the empirical asset pricing literature, we consider six state variables in our empirical model specifications: dividend yield (*DIV*), three-month T-bill rate (*TB*), term spread (*TERM*), default spread (*DEF*), and the Fama and French (1993) firm-size (*SMB*) and book-to-market (*HML*) risk factors. The macroeconomic variables are known to have predictive power for expected stock returns and are widely used in the return predictability literature (Fama and Schwert, 1977; Keim and Stambaugh, 1986; Campbell, 1987; Campbell and Shiller, 1988; Fama and French, 1988, 1989). The dividend yield at daily frequency is defined as the difference between the CRSP value-weighted return including dividends and the same return excluding dividends. The term spread is the difference between yields of the 10-year and 1-year Treasury bonds; daily term spread is available from January 1962. The default spread is defined as the difference between yields of Moody's BAA and

AAA corporate bonds; daily default spread is available from January 1986.[7] The three-month T-bill rate in daily frequency is available from January 1954. The data for macroeconomic variables are taken from Federal Reserve Economic Data (FRED). We also use the risk factors formed by Fama and French (1993) as proxies for investment opportunity. One possible interpretation of the outstanding performance of the Fama–French (1993) three-factor model in explaining the cross-section of stock returns is that the firm-size and book-to-market risk factors are state variables in the context of the Merton's (1973) ICAPM. Liew and Vassalou (2000) document supporting evidence that *SMB* and *HML* contain significant information about future GDP growth. The Fama–French (1993) risk factors are obtained from Kenneth French's website.[8]

To investigate whether the higher-order moment terms introduced in our theoretical model are priced in the cross-section of stock returns, we employ both portfolio sorts and the cross-sectional regressions approach. Using portfolio sorts, we form quintile and decile portfolios based on univariate sorts of each co-moment estimated using daily returns of sample stocks in each year. Then, we compute both value-weighted and equal-weighted monthly returns of portfolios and investigate whether considerable return dispersion is produced across sorted portfolios.

Using cross-sectional regressions, we directly estimate market prices of co-moment risks. We employ the traditional two-pass estimation approach pioneered by Black, Jensen, and Scholes (1972) and Fama and MacBeth (1973) with a couple of modifications. First, co-moments of each asset are estimated by method-of-moment estimation using time-series data in the first stage, since we do not assume a factor model as a return generating process and so cannot apply the conventional time-series regressions. We instead estimate annual co-moments for each asset as the sample moments calculated with daily observations for each year. Second, we do not create test portfolios and use the entire universe of individual stocks as test assets. The original motivation for using portfolios in cross-sectional regression tests is to reduce the errors-in-variables problem in the second stage caused by the regressors

[7] Throughout the paper, we do not report empirical results based on default spread (*DEF*) as a state variable to focus on results for two sub-periods, the pre- and post-1962 periods. For the period 1986 to 2012, the same results as reported in all tables, including those based on default spread (*DEF*), are available upon request.

[8] The Kenneth French's website address is http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/.

estimated in the first stage. However, portfolio formation cannot eliminate the problem completely, and correction is still necessary in calculating standard errors of the second-stage estimates to reflect the estimation errors in the first-stage estimates. Moreover, the recent work of Ang, Liu, and Schwarz (2010) documents that using individual assets leads to more precise estimates in the second stage than using portfolios, because it does not destroy information on cross-sectional dispersion of betas.

More specifically, we estimate co-moments of each asset using daily data for non-overlapping rolling estimation windows at annual frequency. Then, we run cross-sectional regressions of excess returns of stocks on estimated co-moments for each day of the sample periods, and obtain the second-stage estimates for prices of co-moment risks as time-series averages of cross-sectional coefficients, following the method of Fama and MacBeth (1973). To obtain the standard errors of the second-stage estimates corrected for estimation errors in the first stage, we derive an analytic formula of asymptotic standard errors using the general GMM framework described in Cochrane (2005), which generalizes the conventional method proposed by Shanken (1992). Note that the Shanken's (1992) correction cannot be applied to our model, since our model estimates not only the mean and variance, but also the third- and fourth-order moments, unlike general beta pricing models, and hence it cannot be stated as a factor model. The formula for asymptotic standard errors is given in Appendix B.

The estimated model specification is as follows:

$$E_t\left[R_{i,t+1} - R_{ft}\right] = \lambda_0 + \lambda_1 \cdot \beta_{i,t} + \lambda_2 \cdot \gamma_{i,t} + \lambda_3 \cdot \delta_{i,t} + \lambda_4 \cdot \eta_{i,t} + \lambda_5 \cdot \kappa_{i,t} + \lambda_6 \cdot \pi_{i,t} \ , \qquad (10)$$

where co-moments of each asset are defined as

$$\begin{aligned}
\beta_{i,t} &= E_t\left[(R_{i,t+1} - \mu_t)(R_{t+1}^M - \mu_t^M)\right] , \\
\gamma_{i,t} &= E_t\left[(R_{i,t+1} - \mu_t)(R_{t+1}^M - \mu_t^M)^2\right] , \\
\delta_{i,t} &= E_t\left[(R_{i,t+1} - \mu_t)(R_{t+1}^M - \mu_t^M)^3\right] , \\
\eta_{i,t} &= E_t\left[(R_{i,t+1} - \mu_t)(z_{t+1} - \mu_t^z)\right] , \\
\kappa_{i,t} &= E_t\left[(R_{i,t+1} - \mu_t)(R_{t+1}^M - \mu_t^M)(z_{t+1} - \mu_t^z)\right] , \\
\pi_{i,t} &= E_t\left[(R_{i,t+1} - \mu_t)(R_{t+1}^M - \mu_t^M)^2(z_{t+1} - \mu_t^z)\right] .
\end{aligned}$$

We do not standardize co-moments of assets in the units of those of the market portfolio in estimating models. With this specification, the cross-sectional regression coefficients λ_j are directly interpreted as parameters representing investors' risk attitudes, defined in Section 2.2.

3.2 Co-moment Premiums

To examine whether the difference in co-moments earns a significant return premium, we form portfolios of sample stocks based on each co-moment estimated using daily returns in each year. Specifically, annual co-moments are estimated as the sample moments using daily data for stock returns, the market portfolio returns, and state variables. At the beginning of each year, we form quintile and decile portfolios sorted on each of the estimated co-moments, and monthly equal- and value-weighted returns are calculated. Table 1 reports mean monthly returns of co-moment–sorted portfolios over the full sample period.

During the period 1926 to 2012, stocks with high systematic variance β earn higher returns than stocks with low β, indicating that systematic variance is positively priced, as predicted by the traditional CAPM. The mean value-weighted return of the zero-investment portfolio based on decile sorts is 0.76% with a t-statistic of 2.36, and the equal-weighted return is 1.61%, with a t-statistic of 5.39. Differences in systematic skewness γ and systematic kurtosis δ also yield significant return premiums. Focusing on value-weighted strategies, returns of the γ-sorted portfolios decrease monotonically from 1.27% to 0.51% per month, yielding a significant return difference of -0.77% (t-statistic=-3.76) on the zero-investment portfolio. Returns on the δ-sorted portfolio show monotonically increasing patterns from 0.63% to 1.24% per month, with a return premium of 0.61% (t-statistic=2.49). These patterns are consistent with theoretical predictions that systematic skewness is negatively priced and systematic kurtosis is positively priced if risk-averse investors have positive relative prudence and temperance. They are also consistent with the empirical results of Harvey and Siddique (2000) and Dittmar (2002).[9]

[9] The three-moment CAPM of Kraus and Litzenberger (1976) predicts that the price of systematic skewness should have the opposite sign as the skewness of market returns, when systematic skewness is defined as a standardized third moment. Given the fact that aggregate stock returns are negatively skewed during our sample period, our empirical results are also consistent with them.

To investigate the effect of co-moments that reflect intertemporal hedging demands on stock returns, we choose three state variables, the dividend yield (*DIV*), size (*SMB*), and value (*HML*) factors by Fama and French (1993), which are available for the full sample period. For dividend yield, the second- and third-order co-moments with respect to *DIV*, η_{DIV} and κ_{DIV}, do not generate clear patterns of sorted portfolio returns. Returns of quintile portfolios sorted on the fourth-order co-moment π_{DIV}, however, increase monotonically from 0.60% to 1.02%, yielding monthly return premiums of 0.41% (*t*-statistic=2.70) and 0.63% (*t*-statistic=3.37) based on quintile and decile sorts, respectively. The positive return premium of the zero-investment portfolio indicates that an investor's prudence tends to decrease as dividend yield increases, resulting in a reward of high returns on stocks with high π_{DIV}. The sign of the price of π_{DIV} suggests that dividend yield is a proxy for a state favorable for investors, in that an increase in dividend yield corresponds to a decrease in the precautionary savings motive.

For *SMB* and *HML*, none of the co-moments other than the third-order moment with respect to *SMB* show significant return premiums on sorted portfolios. The value-weighted returns of portfolios sorted on κ_{SMB} decrease monotonically, and the zero-investment portfolios based on quintile and decile sorts earn significant returns of -0.67% (*t*-statistic=-3.54) and -0.90% (*t*-statistic=-3.84) per month, respectively. The negative return on the zero investment portfolio means that an investor's risk aversion decreases as *SMB* increases, leading to higher returns required on stocks with low κ_{SMB}, that is, stocks with relatively low systematic risk when *SMB* is high. The sign of the return premium of κ_{SMB} can be well explained by the results of Liew and Vassalou (2000), which document that *SMB* predicts future macroeconomic growth (i.e., good states), combined with the conditional asset pricing literature, which states that risk aversion is high in bad times.[10]

Table 2 and Table 3 report returns of portfolios sorted on co-moments for the sub-periods of 1926 to 1961 and 1962 to 2012, respectively. Focusing on pricing of systematic moments with the market returns, β, γ, and δ, are priced significantly in both the pre- and post-1962

[10] For theoretical work, the model with external habit formation by Campbell and Cochrane (1999) predicts counter-cyclical risk aversion by assuming that risk aversion is governed by the surplus consumption ratio. For empirical work, Lettau and Ludvigson (2001a,b) argue that the consumption-wealth ratio, *cay*, is high when risk or risk aversion is high and document supporting empirical evidence.

periods, though the magnitudes of return premiums earned by the zero-investment portfolios are reduced after 1962. For example, the value-weighted strategies based on the γ-sorted decile yield -1.23% (t-statistic=-2.71) in the pre-1962 period and -0.78% (t-statistic=-2.42) in the post-1962 period. In Table 2, returns of the π_{DIV}- and κ_{SMB}-sorted portfolios generate insignificant premiums during the period 1926 to 1961, though the signs of the premiums are not changed. The results in Table 1 for the full period that π_{DIV} and κ_{SMB} are significantly priced come from strong patterns in Table 3 for the period 1962 to 2012. Examining the sub-periods separately, portfolios sorted on the second-order co-moment with respect to *DIV*, η_{DIV}, and on the fourth-order co-moment with respect to *HML*, π_{HML}, show clear increasing patterns of quintile returns in the pre-1962 period. The value- and equal-weighted returns of the zero-investment portfolio sorted on π_{HML}-quintile are 0.96% and 1.31% with t-statistics of 2.48 and 3.92, respectively, during the pre-1962 period. The sign of the price of π_{HML} implies that *HML* can serve as a proxy for a favorable state with low investor prudence during that period. However, the signs of return premiums produced by η_{DIV} and π_{HML} are reversed after 1962 in Table 3. During the post-1962 period, no co-moments yield clear patterns except for π_{DIV} and κ_{SMB}, while some (κ_{HML} and π_{HML}) earn significant premiums only based on equal-weighted strategies. For state variables of the T-bill rate (*TB*) and the term spread (*TERM*), which are only available for the post-1962 period, none of the portfolios sorted on co-moments with respect to *TB* and *TERM* show dispersion in returns. In results for the period 1986 to 2012 (not tabulated), portfolios sorted on co-moments with respect to the default spread (*DEF*) also do not show clear patterns.

In sum, we find evidence that systematic skewness and kurtosis, as well as variance, are priced and the directions of their prices are consistent with theoretical predictions. When risk-averse investors consistently prefer higher skewness and lower kurtosis, low systematic skewness and high systematic kurtosis of risky assets should require compensation in the form of high returns. In addition, we find that the third- and fourth-order co-moments of stocks with market returns and several state variables are also priced, suggesting that investors have further demands to hedge against intertemporal deterioration of investment opportunities.

Though we find empirical evidence of co-moment premiums, the results from portfolio sorts are not sufficient, because we examine only portfolios based on one-way sorts of each co-moment. Though multivariate portfolio sorts might give clearer evidence, we do not employ them since we consider higher-order moments up to the fourth-order of various state variables, resulting in large numbers of cases of multivariate sorts. Instead, we take a closer look at pricing of co-moments using the cross-sectional regressions approach and directly estimate prices of co-moments in the next subsection.

3.3 Cross-Sectional Regressions

Table 4 presents estimation results of the two-pass cross-sectional regressions specified in equation (10) for the full sample period of 1926 to 2012. We report both *t*-statistics using Fama and MacBeth (1973) standard errors and the standard errors corrected for the errors-in-variables for comparison. Examining the results of static models, cross-sectional regressions of the traditional CAPM (model 1) and the three-moment CAPM (model 2) give statistically significant estimates, while the price of systematic kurtosis is not significant in the four-moment CAPM (model 3). The price of systematic risk β is 5.62 with a *t*-statistic of 4.70 in model 1 and 4.49 with a *t*-statistic of 3.31 in model 2. The estimated price of systematic skewness γ is -603.90 with a *t*-statistic of -4.66. The signs of estimated coefficients are consistent with assumptions on investor preference and previous results by portfolio sort. On the other hand, the coefficient on systematic kurtosis is not statistically significant, nor does it have the predicted sign in model 3. While higher δ earns a higher return based on the univariate sort in Section 3.2, δ is not significantly priced in the presence of the second- and third-order moments. This result implies that investors' relative temperance is not distinguishable from zero, while their relative risk aversion and prudence have nonzero positive values.

With regard to intertemporal models, estimation results are quite different among models, depending on the choice of state variable. When we choose dividend yield (*DIV*) as a state variable, none of the co-moments with respect to *DIV* are significantly priced, while the estimated coefficients of systematic moments with the market returns are not changed substantially. This result is inconsistent with the finding in Section 3.2 that the fourth-order co-moment π_{DIV} generates a significant return dispersion, indicating the effect disappears when systematic risk and skewness are included together. For *SMB*, the second-order co-moment with *SMB*, η_{SMB}, has positive and significant coefficients in models 7, 8, and 9, as expected from empirical studies of Fama and French (1993). The positive price of η_{SMB} indicates that the marginal utility of wealth decreases as *SMB* increases, which results in higher expected returns on stocks with high η_{SMB}. However, the third- and fourth-order moments are not significant. Including *HML* as a state variable does not provide supporting evidence on pricing of co-moments with respect to *HML*. Considering *SMB* and *HML* together in models 13, 14, and 15 does not change substantially the results of models with a single state variable. Overall, significant return premiums produced by portfolios sorted on a couple of higher-order co-moments with state variables become insignificant in the presence of systematic variance and skewness during the period 1926 to 2012. One notable thing is that systematic skewness γ is highly significant in all specifications considered, and the significance is not eliminated by inclusion of other variables. This result implies that investors are likely to have strong motives for precautionary savings and that mean–variance models such as the traditional CAPM cannot capture the effect of such preference on asset prices.

The results of cross-sectional regressions during the period 1926 to 1961 are reported in Table 5 and are very similar to those of the full period. Overall, stocks' systematic variance and covariance with *SMB* are priced positively, and systematic skewness has negative and significant coefficients in all specifications. The estimation results for the post-1962 period yield stronger evidence on the effects of co-moments with state variables on asset prices. Maintaining focus on co-moments with state variables in Table 6, the covariance with *SMB* has positive and significant coefficients in all specifications, as in Table 4 and Table 5. In addition, several higher-order moments are also significantly priced during the post-1962 period; the third-order co-moments with respect to *HML* and *TERM*, κ_{HML} and κ_{TERM}, have negative and statistically significant coefficients in model 11 and model 20. The coefficient of κ_{HML} has a *t*-statistic of -2.12 when standard errors are corrected for the errors-in-variables, and remains significant, though marginal, when we employ two state variables, *SMB* and

HML, together (in model 14). The *t*-statistic of -2.40 for κ_{TERM} also shows that the third-order effect of the term spread is strong. The negative prices of κ_{HML} and κ_{TERM} imply that *HML* and *TERM* are likely proxies for a state with low risk aversion, which leads to higher expected returns on stocks with low κ_{HML} or κ_{TERM} as a reward for higher systematic risk when risk aversion rises. On the other hand, the fourth-order co-moment with respect to *SMB*, π_{SMB}, is marginally significant and has a positive price in model 9 and model 15.

Combining the results from the cross-sectional regression analyses, the pricing effects of higher-order co-moments seem to depend on the choice of a state variable and sample periods. Further, when compared to results in Section 3.2, the magnitudes and directions of return premiums are not retained by the inclusion of other moments. Nevertheless, our results represent strong evidence that systematic skewness is negatively priced, consistent with theoretical predictions and previous empirical studies. Additionally, we find empirical evidence supporting our theoretical model, in that co-moments of higher order than the variance with some state variables could earn significant return premiums. This finding is not investigated in previous studies, and provides a new interpretation of the relation between various state variables and asset prices based on theory. For example, we find that the third-order co-moment with respect to *HML* or *TERM* has a negative effect on stock returns empirically, and this result can be interpreted as that the book-to-market factor and the term spread are relevant in asset pricing due to their relations to variations in risk aversion. Given that the choice of state variables is critical to empirical performance of the model, even though this choice is not based on theory, further studies to find a more pertinent proxy describing changes in investment opportunities are needed to advance understanding of the higher-order effects of intertemporal hedging demands on asset pricing.

4. Implications

In this section, we discuss the prediction of the four-moment ICAPM and its implications for asset pricing anomalies. The model's prediction can be interpreted in two ways—time-series and cross-sectional relations of expected returns with co-moments. As a time-series implication, we examine a mean-variance-skewness trade-off based on risk-attitude parameters implied by the estimated models, extending the literature on risk-return trade-off. As a cross-sectional implication, we investigate relations between returns and co-moments of portfolio strategies based on several cross-sectional anomalies—firm size, book-to-market ratio, momentum, idiosyncratic volatility, and failure probability.

4.1 Intertemporal Relation of Expected Returns, Risk, and Skewness

In the estimation results of cross-sectional regressions, one clear finding is that coefficients on systematic variance β are significantly positive in most specifications with the second- and third-order moments. This could be promising evidence for both our theory and our empirical approach, given that poor empirical performance of the CAPM is well known and that numerous empirical studies have failed to find a positive risk-return trade-off (Campbell, 1987; Breen et al., 1989; Nelson, 1991; Glosten et al., 1993; Harvey, 2001).[11] Moreover, in all the third-order specifications, coefficients on systematic skewness γ are significantly negative, implying that there is an intertemporal relation among mean, variance, and skewness, or a risk-return-skewness trade-off. In other words, at any given level of variance, there is a trade-off between expected returns and negative skewness, as well as a trade-off between expected returns and variance at a given level of skewness. The asset pricing equation from the four-moment ICAPM derived in Section 2 shows that such a relation should exist if the representative agent is risk-averse and prudent.

In this regard, we examine risk-attitude parameters, relative risk aversion (RRA) and relative prudence (RPR), implied by the estimated model in Section 3.3.[12] Specifically, we obtain implied RRA and RPR using the estimated coefficients of cross-sectional regressions, from the following relation:

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[11] Recently, several empirical studies find a significantly positive risk-return trade-off by introducing a new approach. Ghysels et al. (2005) introduce a new variance estimator with past daily squared returns, Guo and Whitelaw (2006) decompose the expected returns into the risk and hedge components, and Bali (2008) use a large cross-section instead of using the market portfolio alone.

[12] We do not consider the relative temperance parameter, or a mean-variance-skewness-kurtosis relation, since the estimated coefficients on systematic kurtosis are generally not positive nor statistically significant.

$$E_t\left[R_{t+1}-R_{ft}\right]=i_t\cdot\gamma_t\cdot E_t\left[(R_{t+1}-\mu_t)(R^M_{t+1}-\mu^M_t)\right]$$
$$-\frac{i_t^2}{2}\cdot\gamma_t\cdot\theta_t\cdot E_t\left[(R_{t+1}-\mu_t)(R^M_{t+1}-\mu^M_t)^2\right]+\cdots\ , \qquad (11)$$

where γ_t is RRA, θ_t is RPR, i_t is the ratio of risky investments to the expected payoff of total investments in the next period, defined as $i_t = w\big/\left\{(1-w)R_{ft}+w\mu_t^M\right\}$, and w denotes the proportion of risky investments in total investments, $W_t - C_t$. To calculate i_t for given w, we use mean annual returns of the market portfolio and the risk-free asset during the sample period. Since there is no criterion to set a value of w, we let w vary within a reasonable range. We assume that w has a positive value, because negative w means that the representative investor has a short position in the market portfolio, which seems implausible. Also, we allow w to have a value greater than 1, which means that the investor invests in risky assets with borrowed money at a risk-free rate. In Table 7, we present implied RRA and RPR for models specified with up to the third-order moments and varying values of w from 0.25 to 2.

Panel A in Table 7 shows implied RRA and RPR for the full sample period of 1926 to 2012. Several features of RRA are notable. First, the implied RRA has a positive value in all considered specifications, with a range of 0.99 to 24.01 and an average of 5.82. This range includes the reasonably acceptable levels of 2 to 5 in the literature. Second, the implied RRA has a smaller value in the third-order specifications than in the second-order specifications. This indicates that taking the third-order moments into consideration improves the estimation of a risk-return trade-off, in that an implied value of RRA matches up better with economic intuition. Third, implied RRA tends to be smaller in intertemporal models than in static models, although there is an exception—intertemporal models with *DIV* as a state variable. This suggests that inclusion of co-moments reflecting intertemporal hedging demands yields a more reasonable relation between risk and return. Finally, implied RRA decreases as the proportion of risky investments, w, increases. This seems natural because the less risk-averse the investor is, the more the investor invests in the risky portfolio.

While there is agreement on the level of RRA from the extensive literature, we can hardly find a general consensus on a reasonable level of RPR. From cross-sectional regressions using daily individual stock returns over the period 1926 to 2012, we obtain an RPR value of 750.76 on average. For w greater than 1, which gives more reasonable RRA values, the average of the implied RPR is 372.13. For RPR, the effect of inclusion of a state variable is not clear—some intertemporal models give smaller values of RPR than a static model, and others do not. Like RRA, implied RPR also decreases as the proportion of risky investments *w* increases. This suggests that the more prudent the investor is, the less the investor invests in risky assets.

Panel B and Panel C in Table 7 show the same results for the periods 1926 to 1961 and 1962 to 2012, respectively. Though overall features of implied RRA and RPR are retained in both periods, the magnitudes of implied parameter values differ slightly between the two sub-periods. Except for intertemporal models with *HML* as a state variable, the implied RRA in the post-1962 period is smaller than that in the pre-1962 period. In contrast, the implied values of RPR in the post-1962 period are relatively larger than those in the pre-1962 period, with the exceptions of models with *SMB* or *HML* as state variables.

To sum up, we investigate a risk-return trade-off and a risk-return-skewness relation implied by the estimation results of cross-sectional regressions in Section 3.3. The implied RRA has a value within a quite reasonable range based on the literature, and it indicates that both our theory and empirical framework work well. In addition, we examine the implied RPR values, which are rarely discussed in previous research. Though there is no general criterion, our empirical findings indicate that there is clear evidence of risk-return-skewness trade-off. This suggests that future research on risk-return trade-off should take the effect of skewness into account as well.

4.2 Co-moment Dispersion and Cross-Sectional Anomalies

Although numerous asset pricing studies have made great efforts to explain anomalous phenomena in financial markets, existing models based on mean–variance portfolio theories leave a number of cross-sectional patterns in stock returns unexplained. We explore the possibility that some cross-sectional anomalies, at least partially, may be attributable to differences in higher-order moments or intertemporal hedging demands. For instance, if portfolios based on a particular anomaly show a decreasing pattern in systematic skewness, which is negatively priced in the cross-section, this implies that part of the abnormal return on the anomaly-based strategy can be due to a reward for accepting low systematic skewness. In this sense, we examine dispersion in co-moments and its relation with returns for several well-known cross-sectional anomalies.

More specifically, we consider five anomalies—the size and value effects documented by Fama and French (1992), the price momentum documented by Jegadeesh and Titman (1993), the idiosyncratic volatility puzzle by Ang, Hodrick, Xing, and Zhang (2006), and the financial distress puzzle in Campbell, Hilscher, and Szilagyi (2008). For the size and value effects, we obtain daily and monthly returns of quintile and decile portfolios sorted on firm size and book-to-market ratio directly from the Kenneth French's website. For momentum portfolios, at the beginning of each month *t*, we construct quintile and decile portfolios based on prior returns over months *t*-7 to *t*-2. Following Ang et al. (2006), we define the idiosyncratic volatility of a stock for month *t* as the standard deviation of residuals from the regression of daily excess returns during month *t* on the Fama–French three factors. Then, at the beginning of each month *t*, we form quintile and decile portfolios sorted on idiosyncratic volatility in month *t*-1. Finally, we construct a monthly measure of failure probability of each stock following the model in Campbell et al. (2008), using quarterly Compustat data. Then, quintile and decile portfolios are formed for each month based on failure probability in month *t*-1. All portfolio returns are value-weighted and obtained at both daily and monthly frequencies. Using daily portfolio returns, we estimate the co-moments of each portfolio as the sample moments during the period. The estimated co-moments and mean monthly returns of portfolios sorted on each of five characteristics are shown in Tables 8 to 12. For better understanding of economic significance, the values of estimated co-moments are represented in the units of corresponding moments of the market returns. We report systematic moments with the market returns in all cases, and report selected second- and third-order co-moments with respect to state variables that show clear and significant dispersed patterns in each case.

Panel A and Panel B of Table 8 report size-sorted portfolio returns and co-moments during the pre- and post-1962 periods, respectively. Since we report only the value-weighted returns, the size effect is relatively weak and the zero-investment portfolio return based on a decile sort is marginally significant, with a *t*-statistic of -1.75 in the pre-1962 period, and even insignificant in the post-1962 period.[13] Nevertheless, returns across quintiles show clear decreasing patterns in both periods. On the other hand, consistent with the fact that the CAPM cannot explain the size premium, systematic variance β displays no clear pattern before 1962, and even an increasing pattern after 1962. Also, both systematic skewness γ and systematic kurtosis δ are not dispersed significantly. Not surprisingly, the size-sorted portfolios have very different levels of covariance with *SMB*, resulting in highly significant η_{SMB} on the zero-investment portfolios in both periods. Both η_{SMB} and η_{HML} clearly decrease from the smallest to the largest quintile, and this demonstrates why the Fama–French three-factor model explains the size effect, given that the prices of *SMB* and *HML* are positive. Focusing on the third-order effects of state variables, κ_{HML} shows significant dispersion, but the negative direction of a premium on κ_{HML} worsens the size effect. The price of κ_{TERM} is also estimated to be significant in Section 3.3, but its pattern across the size quintile is not obvious. Overall, the size premium is well explained by the second-order co-moments with *SMB* and *HML* and not by the higher-order moments.

For portfolios sorted on the book-to-market ratio shown in Table 9, the monthly value premiums are 0.41% (*t*-statistic=2.52) based on a quintile sort and 0.54% (*t*-statistic=2.47) based on a decile sort during the post-1962 period, while the value effect is much weaker during the pre-1962 period. However, systematic variance β displays a decreasing pattern across quintiles during the post-1962 period, while it increases during the pre-1962 period. This indicates that the value premium is made worse by the CAPM. Systematic skewness γ during the post-1962 period decreases from the lowest to the highest book-to-market quintile, yielding negative γ with a *t*-statistic of -2.12 on the zero-investment portfolio. Based on our empirical finding of the negative price of γ, the dispersion in systematic skewness seems to be a clue to the value premium. Dispersion in γ implies that the value stock earns a higher return than the growth stock as compensation for lower systematic skewness. Multiplying the spread in γ between the value and growth portfolios by the price of γ estimated from each specification in Table 6 yields an average monthly premium of 0.10% (0.12%) based on a quintile (decile) sort, which accounts for 25% (23%) of the average value premium during the

[13] The equal-weighted returns on the zero-investment portfolio based on size-sorted decile are -1.60% (*t*-statistic=-2.56) and -0.54% (*t*-statistic=-2.08) per month during the pre- and post-1962 periods, respectively.

post-1962 period. For co-moments related to intertemporal hedging demands, both the covariances with *SMB* and *HML* show clear dispersion in the right direction. This is not surprising given the empirical success of the Fama–French three-factor model in explaining the value effect.

Table 10 shows co-moments and returns of momentum portfolios. Harvey and Siddique (2000) suggest that the momentum effect is related to both systematic and total skewness. They document that past winners have substantially lower skewness than past losers, and argue that the strategy of buying the winner and selling the loser requires bearing substantially negative skewness.[14] In results for the full sample period of 1926 to 2012 (unreported), we find a clear relation between systematic skewness and returns, seemingly consistent with Harvey and Siddique (2000).[15] The mean monthly return on the momentum strategy based on a decile sort is 1.07% (*t*-statistic=3.35) and systematic skewness on the same strategy is -3.33 in the unit of the market skewness (*t*-statistic=-2.65). However, our results of the sub-periods cast doubt on the argument of Harvey and Siddique (2000). Comparing Panel A and Panel B in Table 10, the momentum strategy earns a significantly high return only during the period 1962 to 2012. In contrast, the dispersion in γ is clear and significant in the period before 1962, while it becomes insignificant during the period of the high abnormal return. This is a somewhat surprising finding. Turning to the third-order effects of intertemporal hedging demands, the third-order co-moment with *TERM*, κ_{TERM}, generates a decreasing pattern from the loser to the winner and has a negative and significant value, with a *t*-statistic of -4.99, on the zero-investment portfolio. Given the finding in Section 3.3 that κ_{TERM} is negatively priced during the post-1962 period, which is interpreted as that high *TERM* stands for states of low risk aversion, the dispersion in κ_{TERM} implies that the winner tends to have low κ_{TERM}, or relatively high systematic risk in times of high risk aversion (low *TERM*), and that the loser tends to have high κ_{TERM}, or low systematic risk in times of high risk aversion. The difference in κ_{TERM} between the winner and loser portfolios multiplied by the estimated price of κ_{TERM} in Table 6 generates monthly premiums of 0.71%

[14] Strictly, this is true only when considering systematic skewness, which preserves the linear property. Due to the non-linearity of total skewness, the skewness of the long-short portfolio need not be the same as the difference in skewness between the long- and the short- legs.

[15] The results for the full sample period are available on request.

and 0.86% based on quintile and decile sorts, respectively. Remarkably, these figures amount to 107% and 59% of the average momentum premiums during the post-1962 period. Consequently, we interpret that a large part of the momentum profit is a reward for accepting higher systematic risk when the investor is more risk-averse.

For the idiosyncratic volatility puzzle reported in Table 11, the abnormal return on the zero-investment portfolio is observed during the period after 1962. Though systematic variance, skewness, kurtosis, and covariance with *SMB* show significant dispersion across quintile portfolios, the directions of the dispersion indicate that including them in the asset pricing model makes even worse the idiosyncratic volatility puzzle. On the other hand, the covariance with *HML*, η_{HML}, decreases from the lowest to the highest idiosyncratic volatility quintile during the post-1962 period, and η_{HML} of the zero-investment portfolio based on a quintile sort has a *t*-statistic of -4.21. If we accept the positive price of η_{HML} as shown in numerous empirical studies, this implies that the low-volatility stock has higher covariance with *HML*, meaning that it earns high return when the marginal utility is low (or *HML* is high), than the high-volatility stock. Therefore, the abnormal return on the low-minus-high volatility strategy can be regarded as compensation for cyclical variations.

Finally, Table 12 presents co-moments and returns of portfolios sorted on failure probability. Due to data availability, the portfolio return begins in September 1972. Focusing on the results in Panel A of the full period, we observe that the low return on the zero-investment portfolio buying the high-failure-probability stocks and selling the low-failure-probability stocks mainly comes from the highest failure probability portfolio. Overall patterns in co-moments indicate that the static version of three-moment CAPM and the Fama–French three-factor model, as well as the traditional CAPM, are not at all helpful in explaining the distress puzzle. For the third-order co-moment with *TERM*, κ_{TERM} increases from the lowest to the highest failure probability quintile, generating a positive and significant κ_{TERM} with a *t*-statistic of 2.59 on the zero-investment portfolio. The dispersion in κ_{TERM} between the lowest and the highest failure probability quintile, multiplied by the estimated price of κ_{TERM} in Table 6, yields a monthly premium of 0.55%, which makes up 82% of the average return difference between the lowest and highest quintiles. Like the momentum portfolio, we interpret that the low failure probability stock tends to have low

κ_{TERM}, or high systematic risk when relative risk aversion is high (low *TERM*), whereas the high failure probability stock has high κ_{TERM}, or low systematic risk when relative risk aversion is high. Therefore, the required return on the low failure probability stock is relatively higher to bear higher systematic risk in times of high risk aversion.

To summarize, the cross-sectional implication of the four-moment ICAPM is that dispersion in higher-order co-moments, as well as dispersion in systematic risk, is related to difference in returns. We explore the relation between co-moment dispersion and returns based on several cross-sectional anomalies, and find evidence that abnormal returns on anomaly strategies can be explained partially by the risk-based story. Specifically, we find that the value stock has lower systematic skewness than the growth stock, the low-volatility stock has higher covariance with *HML* than the high-volatility stock, and the winner and the low-failure-probability stock have the third-order co-moment with *TERM* lower than the loser and the high-failure-probability stock. These findings indicate a direction for future research in that the cross-sectional anomalies could be explained by a risk-return-skewness trade-off.

5. Conclusion

Traditionally, most asset pricing theories have been developed based on only mean and variance of asset returns, despite the stylized fact that stock returns are both skewed and fat-tailed. Based on preference for positive skewness and low kurtosis, several papers have taken the effect of higher-order moments into account in static asset pricing models, and find evidence that systematic skewness and kurtosis play a role in explaining the cross-section of stock returns. We extend the literature by incorporating the effect of stochastic investment opportunities as well as higher-order moments on expected stock returns, and by presenting a new estimation procedure to overcome the errors-in-variables problem that may lead to huge estimation errors in higher moment models. We propose a four-moment intertemporal asset pricing model that encompasses the traditional, three- and four-moment static CAPM and the intertemporal CAPM based on mean–variance optimization. In comparison with existing models, our model takes intertemporal hedging demands of the long-term investor into account as well, and thereby further captures compensation for unfavorable changes in systematic risk and systematic skewness in expected returns.

We also develop empirical support for the theoretical model using U.S. stock market data. With daily returns of individual stocks and various state variables as predictors for investment opportunities, we investigate whether co-moments in the derived asset pricing equation are priced in the cross-section. Based on both portfolio sorts and cross-sectional regressions, we find strong evidence that systematic skewness is negatively priced. Moreover, we find that the third-order co-moment with the market return and each state variable has a significantly negative price when the state variable predicts good states. We interpret this result as that such state variables, the term spread and the *HML* factor, are related to variations in risk aversion. However, most of the fourth-order co-moment premiums observed from univariate portfolio sorts disappear in the presence of systematic variance and systematic skewness.

Based on the estimation result of the cross-sectional regressions, we further explore implications of the four-moment ICAPM for asset pricing anomalies. As a time-series implication, we examine relative risk aversion and relative prudence implied by the estimated models and confirm that there is a risk-return-skewness relation; at a given level of variance, there is a trade-off of expected returns and negative skewness. As a cross-sectional implication, we investigate whether dispersion in co-moments may be related to cross-sectional patterns in stock returns for several well-known anomalies. The results suggest that, at least partially, the value premium can be a reward for bearing lower systematic skewness, and that the abnormal returns for strategies based on momentum and failure probability can be due to compensation for accepting higher systematic risk associated with high risk aversion.

Overall, investors consistently prefer positive skewness and there is a significant relation of expected returns, risk, and skewness, while the evidence on kurtosis preference is not clear. Our findings provide a clue to future asset pricing research on the role of higher-order co-moments in resolving unexplained anomalies.

Appendix A. Derivation of Equilibrium Expected Returns in Equation (6)

Equation (5) can be rearranged as follows:

$$\mu_0 - \mathbf{1}_N R_{f0} = -\frac{2}{I}\cdot\frac{G_v}{G_m}\cdot\Sigma_0 V - \Phi_0\cdot\frac{G_h'}{G_m} - \frac{3}{I^2}\cdot\frac{G_s}{G_m}\cdot\Theta_0(V\otimes V) - \frac{2}{I}\cdot\Psi_0(V\otimes I_K)\cdot\frac{G_l'}{G_m}$$
$$-\frac{4}{I^3}\cdot\frac{G_k}{G_m}\cdot\Pi_0(V\otimes V\otimes V) - \frac{3}{I^2}\cdot\Omega_0(V\otimes V\otimes I_K)\cdot\frac{G_n'}{G_m}\ . \qquad \text{(A1)}$$

Note that:

$$\begin{aligned}\Sigma_t V &= Cov_t(R_{t+1},R_{t+1})V = Cov_t(R_{t+1},V'R_{t+1}) = (\mathbf{1}_N'V)\cdot Cov_t(R_{t+1},R_{t+1}^M)\\ &= (\mathbf{1}_N'V)\cdot E_t\left[(R_{t+1}-\mu_t)(R_{t+1}^M-\mu_t^M)\right]\end{aligned},$$

where $R_{t+1}^M = (\mathbf{1}_N'V)^{-1}V'R_{t+1}$ is gross return of the value-weighted portfolio of the *N* risky assets, $\mu_t^M = E_t(R_{t+1}^M) = (\mathbf{1}_N'V)^{-1}V'\mu_t$ is the mean return of the value-weighted portfolio.

$$\begin{aligned}\Theta_t(V\otimes V) &= E_t\left[(R_{t+1}-\mu_t)(R_{t+1}-\mu_t)'\otimes(R_{t+1}-\mu_t)'\right](V\otimes V)\\ &= E_t\left[(R_{t+1}-\mu_t)(R_{t+1}-\mu_t)'V\otimes(R_{t+1}-\mu_t)'V\right]\\ &= (\mathbf{1}_N'V)^2\cdot E_t\left[(R_{t+1}-\mu_t)(R_{t+1}^M-\mu_t^M)^2\right]\end{aligned},$$

$$\begin{aligned}\Pi_t(V\otimes V\otimes V) &= E_t\left[(R_{t+1}-\mu_t)(R_{t+1}-\mu_t)'\otimes(R_{t+1}-\mu_t)'\otimes(R_{t+1}-\mu_t)'\right](V\otimes V\otimes V)\\ &= E_t\left[(R_{t+1}-\mu_t)(R_{t+1}-\mu_t)'V\otimes(R_{t+1}-\mu_t)'V\otimes(R_{t+1}-\mu_t)'V\right]\\ &= (\mathbf{1}_N'V)^3\cdot E_t\left[(R_{t+1}-\mu_t)(R_{t+1}^M-\mu_t^M)^3\right]\end{aligned},$$

$$\Phi_t = Cov_t(R_{t+1},z_{t+1}) = E_t\left[(R_{t+1}-\mu_t)(z_{t+1}-E_t(z_{t+1}))'\right],$$

$$\begin{aligned}\Psi_t(V\otimes I_K) &= E_t\left[(R_{t+1}-\mu_t)(R_{t+1}-\mu_t)'\otimes(z_{t+1}-E_t(z_{t+1}))'\right](V\otimes I_K)\\ &= E_t\left[(R_{t+1}-\mu_t)(R_{t+1}-\mu_t)'V\otimes(z_{t+1}-E_t(z_{t+1}))'I_K\right]\\ &= (\mathbf{1}_N'V)\cdot E_t\left[(R_{t+1}-\mu_t)(R_{t+1}^M-\mu_t^M)(z_{t+1}-E_t(z_{t+1}))'\right]\end{aligned},$$

$$\begin{aligned}\Omega_t(V\otimes V\otimes I_K) &= E_t\left[(R_{t+1}-\mu_t)(R_{t+1}-\mu_t)'\otimes(R_{t+1}-\mu_t)'\otimes(z_{t+1}-E_t(z_{t+1}))'\right](V\otimes V\otimes I_K)\\ &= E_t\left[(R_{t+1}-\mu_t)(R_{t+1}-\mu_t)'V\otimes(R_{t+1}-\mu_t)'V\otimes(z_{t+1}-E_t(z_{t+1}))'I_K\right]\\ &= (\mathbf{1}_N'V)^2\cdot E_t\left[(R_{t+1}-\mu_t)(R_{t+1}^M-\mu_t^M)^2(z_{t+1}-E_t(z_{t+1}))'\right]\ .\end{aligned}$$

By substituting the preceding formulas into equation (A1), we obtain the equilibrium expected returns in equation (6).

Appendix B. Asymptotic Standard Errors of Two-Pass Cross-Sectional Regressions

Following notation on the general GMM framework in Cochrane (2005), the estimated model can be represented as the following moment conditions.

$$\underset{(8N+2)\times 1}{g_T(b)} = \begin{bmatrix} E_T\left[R_{mt}-\mu_m\right] \\ E_T\left[z_t-\mu_z\right] \\ E_T\left[\mathbf{R}_t-\boldsymbol{\mu}\right] \\ E_T\left[(\mathbf{R}_t-\boldsymbol{\mu})(R_{mt}-\mu_m)-\boldsymbol{\beta}\right] \\ E_T\left[(\mathbf{R}_t-\boldsymbol{\mu})(R_{mt}-\mu_m)^2-\boldsymbol{\gamma}\right] \\ E_T\left[(\mathbf{R}_t-\boldsymbol{\mu})(R_{mt}-\mu_m)^3-\boldsymbol{\delta}\right] \\ E_T\left[(\mathbf{R}_t-\boldsymbol{\mu})(z_t-\mu_z)-\boldsymbol{\eta}\right] \\ E_T\left[(\mathbf{R}_t-\boldsymbol{\mu})(R_{mt}-\mu_m)(z_t-\mu_z)-\boldsymbol{\kappa}\right] \\ E_T\left[(\mathbf{R}_t-\boldsymbol{\mu})(R_{mt}-\mu_m)^2(z_t-\mu_z)-\boldsymbol{\pi}\right] \\ E_T\left[\mathbf{R}_t-\mathbf{1}\lambda_0-\boldsymbol{\beta}\lambda_1-\boldsymbol{\gamma}\lambda_2-\boldsymbol{\delta}\lambda_3-\boldsymbol{\eta}\lambda_4-\boldsymbol{\kappa}\lambda_5-\boldsymbol{\pi}\lambda_6\right] \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \mathbf{0}_N \\ \mathbf{0}_N \\ \mathbf{0}_N \\ \mathbf{0}_N \\ \mathbf{0}_N \\ \mathbf{0}_N \\ \mathbf{0}_N \\ \mathbf{0}_N \end{bmatrix},$$

where R_{mt}, z_t, and $\mathbf{R}_t$ are excess returns of the market portfolio, state variables, and excess returns of N individual assets, and μ_m, μ_z, and $\boldsymbol{\mu}$ are their expected values, respectively, and the vector of estimated parameters, b, is

$$\underset{1\times(7N+9)}{b'} = \begin{bmatrix} \mu_m & \mu_z & \boldsymbol{\mu}' & \boldsymbol{\beta}' & \boldsymbol{\gamma}' & \boldsymbol{\delta}' & \boldsymbol{\eta}' & \boldsymbol{\kappa}' & \boldsymbol{\pi}' & \lambda_0 & \lambda_1 & \lambda_2 & \lambda_3 & \lambda_4 & \lambda_5 & \lambda_6 \end{bmatrix}.$$

Note that the first $(2 + N)$ rows in $g_T(b)$ are included to recognize sampling variation induced by the fact that the mean of factors (market returns and state variables) and excess returns are estimated. The next $6N$ rows represent first-stage estimation of co-moments, and the last N rows represent second-stage cross-sectional regressions.

The matrix that defines which moment conditions are set to zero is

$$\underset{(7N+9)\times(8N+2)}{a} = \begin{bmatrix} \mathbf{I}_{7N+2} & \mathbf{0}_{(7N+2)\times N} \\ \mathbf{0}_{7\times(7N+2)} & \boldsymbol{\theta}' \end{bmatrix},$$

where $\underset{N\times 7}{\boldsymbol{\theta}} = \begin{pmatrix} \mathbf{1} & \boldsymbol{\beta} & \boldsymbol{\gamma} & \boldsymbol{\delta} & \boldsymbol{\eta} & \boldsymbol{\kappa} & \boldsymbol{\pi} \end{pmatrix}$.

The matrix of derivatives of the moment conditions with respect to the parameter is

$$\underset{(8N+2)\times(7N+9)}{d} = \frac{\partial g_T}{\partial b'}$$

$$= \begin{bmatrix} -1 & 0 & \mathbf{0}'_N & \mathbf{0}'_N & \mathbf{0}'_N & \mathbf{0}'_N & \mathbf{0}'_N & \mathbf{0}'_N & \mathbf{0}'_N & \mathbf{0}'_7 \\ 0 & -1 & \mathbf{0}'_N & \mathbf{0}'_N & \mathbf{0}'_N & \mathbf{0}'_N & \mathbf{0}'_N & \mathbf{0}'_N & \mathbf{0}'_N & \mathbf{0}'_7 \\ \mathbf{0}_N & \mathbf{0}_N & -\mathbf{I}_N & \mathbf{0}_{N\times N} & \mathbf{0}_{N\times N} & \mathbf{0}_{N\times N} & \mathbf{0}_{N\times N} & \mathbf{0}_{N\times N} & \mathbf{0}_{N\times N} & \mathbf{0}_{N\times 7} \\ \mathbf{0}_N & \mathbf{0}_N & \mathbf{0}_{N\times N} & -\mathbf{I}_N & \mathbf{0}_{N\times N} & \mathbf{0}_{N\times N} & \mathbf{0}_{N\times N} & \mathbf{0}_{N\times N} & \mathbf{0}_{N\times N} & \mathbf{0}_{N\times 7} \\ -2E(\tilde{\mathbf{R}}_t\tilde{R}_{mt}) & \mathbf{0}_N & -\mathbf{I}_N E(\tilde{R}^2_{mt}) & \mathbf{0}_{N\times N} & -\mathbf{I}_N & \mathbf{0}_{N\times N} & \mathbf{0}_{N\times N} & \mathbf{0}_{N\times N} & \mathbf{0}_{N\times N} & \mathbf{0}_{N\times 7} \\ -3E(\tilde{\mathbf{R}}_t\tilde{R}^2_{mt}) & \mathbf{0}_N & -\mathbf{I}_N E(\tilde{R}^3_{mt}) & \mathbf{0}_{N\times N} & \mathbf{0}_{N\times N} & -\mathbf{I}_N & \mathbf{0}_{N\times N} & \mathbf{0}_{N\times N} & \mathbf{0}_{N\times N} & \mathbf{0}_{N\times 7} \\ \mathbf{0}_N & \mathbf{0}_N & \mathbf{0}_{N\times N} & \mathbf{0}_{N\times N} & \mathbf{0}_{N\times N} & \mathbf{0}_{N\times N} & -\mathbf{I}_N & \mathbf{0}_{N\times N} & \mathbf{0}_{N\times N} & \mathbf{0}_{N\times 7} \\ -E(\tilde{\mathbf{R}}_t\tilde{z}_t) & -E(\tilde{\mathbf{R}}_t\tilde{R}_{mt}) & -\mathbf{I}_N E(\tilde{R}_{mt}\tilde{z}_t) & \mathbf{0}_{N\times N} & \mathbf{0}_{N\times N} & \mathbf{0}_{N\times N} & \mathbf{0}_{N\times N} & -\mathbf{I}_N & \mathbf{0}_{N\times N} & \mathbf{0}_{N\times 7} \\ -2E(\tilde{\mathbf{R}}_t\tilde{R}_{mt}\tilde{z}_t) & -E(\tilde{\mathbf{R}}_t\tilde{R}^2_{mt}) & -\mathbf{I}_N E(\tilde{R}^2_{mt}\tilde{z}_t) & \mathbf{0}_{N\times N} & \mathbf{0}_{N\times N} & \mathbf{0}_{N\times N} & \mathbf{0}_{N\times N} & \mathbf{0}_{N\times N} & -\mathbf{I}_N & \mathbf{0}_{N\times 7} \\ \mathbf{0}_N & \mathbf{0}_N & \mathbf{0}_{N\times N} & -\lambda_1\mathbf{I}_N & -\lambda_2\mathbf{I}_N & -\lambda_3\mathbf{I}_N & -\lambda_4\mathbf{I}_N & -\lambda_5\mathbf{I}_N & -\lambda_6\mathbf{I}_N & -\boldsymbol{\theta} \end{bmatrix},$$

and the covariance matrix of the $g_T(b)$ is $S = \sum_{j=-\infty}^{\infty} E\left[g(b)g(b)'\right]$.

The asymptotic variance formula of the general GMM estimate in Cochrane (2005) is

$$\operatorname{var}(\hat{b}) = \frac{1}{T}(ad)^{-1} aSa'(ad)^{-1\prime}$$

$$= \frac{1}{T}\cdot\begin{bmatrix} \underset{(7N+2)\times(7N+2)}{ad_{11}^{-1}} & \underset{(7N+2)\times 7}{\mathbf{0}} \\ \underset{7\times(7N+2)}{-\left(\boldsymbol{\theta}'\boldsymbol{\theta}\right)^{-1}\cdot\boldsymbol{\theta}'K} & \underset{7\times 7}{-\left(\boldsymbol{\theta}'\boldsymbol{\theta}\right)^{-1}} \end{bmatrix}\cdot\begin{bmatrix} \underset{(7N+2)\times(7N+2)}{S_{11}} & \underset{(7N+2)\times 7}{S_{12}\boldsymbol{\theta}} \\ \underset{7\times(7N+2)}{\boldsymbol{\theta}'S_{21}} & \underset{7\times 7}{\boldsymbol{\theta}'S_{22}\boldsymbol{\theta}} \end{bmatrix}\cdot\begin{bmatrix} \underset{(7N+2)\times(7N+2)}{ad_{11}^{-1\prime}} & \underset{(7N+2)\times 7}{-K'\boldsymbol{\theta}\left(\boldsymbol{\theta}'\boldsymbol{\theta}\right)^{-1}} \\ \underset{7\times(7N+2)}{\mathbf{0}} & \underset{7\times 7}{-\left(\boldsymbol{\theta}'\boldsymbol{\theta}\right)^{-1}} \end{bmatrix},$$

where $\underset{N\times(7N+2)}{K} = \begin{bmatrix} \underset{N\times 1}{K_1} & \underset{N\times 1}{K_2} & \underset{N\times N}{K_3} & \lambda_1\mathbf{I}_N & \lambda_2\mathbf{I}_N & \lambda_3\mathbf{I}_N & \lambda_4\mathbf{I}_N & \lambda_5\mathbf{I}_N & \lambda_6\mathbf{I}_N \end{bmatrix}$,

$K_1 = -\lambda_2 2E(\tilde{\mathbf{R}}_t\tilde{R}_{mt}) - \lambda_3 3E(\tilde{\mathbf{R}}_t\tilde{R}^2_{mt}) - \lambda_5 E(\tilde{\mathbf{R}}_t\tilde{z}_t) - \lambda_6 2E(\tilde{\mathbf{R}}_t\tilde{R}_{mt}\tilde{z}_t)$, $K_2 = -\lambda_5 E(\tilde{\mathbf{R}}_t\tilde{R}_{mt}) - \lambda_6 E(\tilde{\mathbf{R}}_t\tilde{R}^2_{mt})$,

$K_3 = -\left\{\lambda_2 E(\tilde{R}^2_{mt}) + \lambda_3 E(\tilde{R}^3_{mt}) + \lambda_5 E(\tilde{R}_{mt}\tilde{z}_t) + \lambda_6 E(\tilde{R}^2_{mt}\tilde{z}_t)\right\}\mathbf{I}_N$.

The variance of the second-stage estimates, $\boldsymbol{\lambda} = \begin{pmatrix}\lambda_0 & \lambda_1 & \lambda_2 & \lambda_3 & \lambda_4 & \lambda_5 & \lambda_6\end{pmatrix}'$, is the 7x7 lower-right block matrix of $\operatorname{var}(\hat{b})$. It is obtained as follows:

$$\operatorname{var}(\hat{\boldsymbol{\lambda}}) = \frac{1}{T}\left(\boldsymbol{\theta}'\boldsymbol{\theta}\right)^{-1}\cdot\boldsymbol{\theta}'\boldsymbol{\Omega}\boldsymbol{\theta}\left(\boldsymbol{\theta}'\boldsymbol{\theta}\right)^{-1},$$

where $\underset{N\times N}{\boldsymbol{\Omega}} = ESE'$, $\underset{N\times(8N+2)}{E} = \begin{bmatrix} \underset{N\times 1}{K_1} & \underset{N\times 1}{K_2} & \underset{N\times N}{K_3} & \lambda_1\mathbf{I}_N & \lambda_2\mathbf{I}_N & \lambda_3\mathbf{I}_N & \lambda_4\mathbf{I}_N & \lambda_5\mathbf{I}_N & \lambda_6\mathbf{I}_N & \mathbf{I}_N \end{bmatrix}$, and K_1, K_2, and K_3 are defined as above.

In calculating the matrix S, we assume the errors are i.i.d. and market returns and state variables are uncorrelated over time.

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Table 1. Portfolios sorted on co-moments: Full sample period (1926–2012)

This table reports mean of co-moments and mean monthly returns of portfolios sorted on co-moments of sample stocks. Annual co-moments are estimated as the sample moments using daily data of stock returns, market portfolio returns, and state variables. At the beginning of each year, we form quintile and decile portfolios sorted on each of the estimated co-moments, and monthly equal-weighted (EW) and value-weighted (VW) returns are calculated. β is covariance with the market returns, γ is co-skewness with the market returns, and δ is co-kurtosis with the market returns. η is covariance with a state variable, κ is co-skewness with the market returns and a state variable, and π is co-kurtosis with the squared market returns and a state variable. State variables are the dividend yield (*DIV*), size (*SMB*) and value (*HML*) factors of Fama and French (1993). Each column of "1" to "5" indicates quintile portfolios in ascending order, and the column "5-1" ("10-1") indicates a zero-investment portfolio that is long in the highest moment quintile (decile) and short in the lowest moment quintile (decile). The mean values of co-moments are shown in the units of corresponding moments of the market returns. The numbers in parentheses are *t*-statistics adjusted using Newey–West (1987) standard errors with 12 lags. The sample period is January 1926 to December 2012.

		1	2	3	4	5	5-1	10-1
β	Mean	0.121	0.523	0.831	1.180	1.815	1.694	2.122
	EW	0.706	0.968	1.166	1.384	1.897	1.191	1.613
		(3.878)	(4.697)	(5.036)	(4.984)	(5.113)	(4.958)	(5.394)
	VW	0.729	0.776	0.970	0.993	1.183	0.453	0.755
		(5.702)	(5.795)	(5.744)	(4.754)	(3.758)	(1.809)	(2.357)
γ	Mean	-8.515	-3.316	-1.414	0.840	5.158	13.673	18.308
	EW	1.737	1.341	1.192	1.038	0.787	-0.950	-1.132
		(5.438)	(5.506)	(5.684)	(4.585)	(2.798)	(-5.448)	(-5.355)
	VW	1.274	1.159	1.037	0.871	0.509	-0.765	-0.968
		(4.678)	(5.874)	(6.187)	(4.672)	(2.294)	(-3.762)	(-3.607)
δ	Mean	0.039	0.521	0.861	1.254	2.020	1.982	2.535
	EW	0.689	1.027	1.160	1.386	1.855	1.167	1.494
		(3.477)	(5.018)	(5.034)	(5.107)	(5.092)	(5.208)	(5.494)
	VW	0.630	0.772	0.917	1.125	1.241	0.611	0.917
		(4.402)	(5.596)	(5.357)	(5.133)	(3.876)	(2.492)	(2.865)
η_{DIV}	Mean	-23.387	-6.997	-1.171	4.286	19.251	42.638	60.180
	EW	1.227	1.252	1.201	1.207	1.227	0.000	0.105
		(4.052)	(5.835)	(5.530)	(5.234)	(4.068)	(0.000)	(0.864)
	VW	0.735	0.935	0.982	0.954	0.851	0.116	0.323
		(2.976)	(5.467)	(5.717)	(4.976)	(3.332)	(0.761)	(1.945)
κ_{DIV}	Mean	-8.987	-2.257	0.306	3.186	9.447	18.434	25.325
	EW	1.281	1.268	1.217	1.222	1.134	-0.147	-0.217
		(4.293)	(5.823)	(5.775)	(5.221)	(3.645)	(-1.137)	(-1.325)
	VW	0.965	0.974	0.939	0.884	0.801	-0.164	-0.116
		(4.325)	(5.830)	(5.694)	(4.477)	(3.086)	(-0.996)	(-0.554)
π_{DIV}	Mean	-8.580	-2.220	-0.137	1.838	7.093	15.673	22.472
	EW	1.076	1.156	1.227	1.255	1.394	0.318	0.353
		(3.812)	(5.338)	(5.745)	(5.246)	(4.410)	(2.828)	(2.467)
	VW	0.603	0.826	0.984	0.993	1.016	0.413	0.626
		(2.720)	(5.068)	(5.896)	(5.015)	(3.854)	(2.699)	(3.369)

(*continued*)

Table 1. (*continued*)

		1	2	3	4	5	5-1	10-1
η_{SMB}	Mean	-1.482	1.031	2.644	6.028	8.647	10.129	13.187
	EW	1.074	1.133	1.240	1.261	1.438	0.364	0.432
		(4.576)	(5.320)	(5.805)	(4.900)	(3.881)	(1.585)	(1.483)
	VW	0.957	0.945	1.014	0.898	0.740	-0.217	-0.257
		(4.766)	(5.898)	(5.557)	(3.723)	(2.275)	(-0.868)	(-0.827)
κ_{SMB}	Mean	-38.919	-12.849	-0.766	11.172	38.578	77.497	103.529
	EW	1.466	1.328	1.227	1.104	0.972	-0.495	-0.552
		(4.547)	(5.639)	(5.606)	(4.910)	(3.460)	(-2.820)	(-2.481)
	VW	1.161	1.118	1.023	0.785	0.489	-0.672	-0.903
		(4.407)	(5.754)	(6.022)	(4.202)	(1.995)	(-3.538)	(-3.841)
π_{SMB}	Mean	-1.636	0.026	0.909	2.169	4.817	6.453	8.959
	EW	1.098	1.163	1.226	1.261	1.368	0.270	0.332
		(4.357)	(5.535)	(5.318)	(4.977)	(4.056)	(1.345)	(1.366)
	VW	0.848	0.948	1.007	0.996	0.802	-0.046	-0.223
		(4.252)	(5.969)	(5.671)	(4.509)	(2.615)	(-0.194)	(-0.734)
η_{HML}	Mean	-4.917	-1.179	0.483	2.071	5.963	10.880	14.665
	EW	1.207	1.114	1.168	1.216	1.442	0.235	0.233
		(4.340)	(4.916)	(5.134)	(4.665)	(4.414)	(0.885)	(0.707)
	VW	0.859	0.837	0.969	1.077	1.097	0.239	0.127
		(4.165)	(5.216)	(5.250)	(4.851)	(3.926)	(0.961)	(0.393)
κ_{HML}	Mean	-20.141	-4.390	1.945	8.980	36.412	56.553	83.458
	EW	1.252	1.195	1.152	1.213	1.299	0.047	0.073
		(4.156)	(5.329)	(5.425)	(5.055)	(4.206)	(0.263)	(0.333)
	VW	0.931	1.008	0.943	0.985	0.910	-0.021	-0.083
		(3.985)	(5.735)	(5.601)	(5.006)	(3.556)	(-0.108)	(-0.320)
π_{HML}	Mean	-1.726	-0.495	0.101	0.735	1.884	3.611	4.861
	EW	1.143	1.124	1.206	1.264	1.388	0.245	0.255
		(4.053)	(4.997)	(5.223)	(4.941)	(4.406)	(1.004)	(0.851)
	VW	0.776	0.804	0.970	1.057	1.122	0.346	0.385
		(3.771)	(5.008)	(5.449)	(4.847)	(4.024)	(1.439)	(1.241)

Table 2. Portfolios sorted on co-moments: Pre-1962 period (1926–1961)

This table reports mean of co-moments and mean monthly returns of portfolios sorted on co-moments of sample stocks. Annual co-moments are estimated as the sample moments using daily data of stock returns, market portfolio returns, and state variables. At the beginning of each year, we form quintile and decile portfolios sorted on each of the estimated co-moments, and monthly equal-weighted (EW) and value-weighted (VW) returns are calculated. β is covariance with the market returns, γ is co-skewness with the market returns, and δ is co-kurtosis with the market returns. η is covariance with a state variable, κ is co-skewness with the market returns and a state variable, and π is co-kurtosis with the squared market returns and a state variable. State variables are the dividend yield (*DIV*), size (*SMB*) and value (*HML*) factors of Fama and French (1993). Each column of "1" to "5" indicates quintile portfolios in ascending order, and the column "5-1" ("10-1") indicates a zero-investment portfolio that is long in the highest moment quintile (decile) and short in the lowest moment quintile (decile). The mean values of co-moments are shown in the units of corresponding moments of the market returns. The numbers in parentheses are *t*-statistics adjusted using Newey–West (1987) standard errors with 12 lags. The sample period is January 1926 to December 1961.

		1	2	3	4	5	5-1	10-1
β	Mean	0.279	0.661	0.983	1.342	1.931	1.652	2.028
	EW	0.774	1.009	1.235	1.462	2.027	1.253	1.628
		(2.378)	(2.578)	(2.774)	(2.704)	(3.053)	(3.267)	(3.512)
	VW	0.696	0.763	1.062	1.282	1.494	0.797	1.127
		(2.882)	(2.913)	(3.054)	(3.055)	(2.523)	(1.952)	(2.081)
γ	Mean	-2.848	-1.459	-0.765	-0.083	1.616	4.464	6.014
	EW	1.863	1.405	1.226	1.106	0.901	-0.962	-1.114
		(3.338)	(2.984)	(3.017)	(2.453)	(1.709)	(-3.349)	(-3.277)
	VW	1.545	1.299	1.117	0.920	0.606	-0.939	-1.230
		(3.350)	(3.481)	(3.482)	(2.391)	(1.426)	(-2.600)	(-2.706)
δ	Mean	0.226	0.671	1.025	1.421	2.130	1.905	2.390
	EW	0.764	1.076	1.168	1.496	1.990	1.227	1.561
		(2.144)	(2.788)	(2.612)	(2.842)	(3.026)	(3.436)	(3.722)
	VW	0.645	0.750	0.974	1.415	1.491	0.847	1.174
		(2.624)	(2.795)	(2.765)	(3.216)	(2.436)	(1.974)	(2.141)
η_{DIV}	Mean	-24.019	-7.799	-1.525	3.892	17.385	41.404	58.361
	EW	1.273	1.268	1.211	1.261	1.472	0.199	0.456
		(2.421)	(3.183)	(2.820)	(2.745)	(2.545)	(1.300)	(2.271)
	VW	0.863	0.889	1.018	1.111	1.150	0.287	0.539
		(2.273)	(2.953)	(3.053)	(2.863)	(2.282)	(1.115)	(2.216)
κ_{DIV}	Mean	-9.063	-2.364	0.288	3.106	9.571	18.634	25.915
	EW	1.466	1.335	1.254	1.220	1.256	-0.211	-0.290
		(2.812)	(3.188)	(3.052)	(2.677)	(2.153)	(-1.117)	(-1.099)
	VW	1.073	1.017	0.952	0.915	1.095	0.022	0.044
		(2.895)	(3.386)	(2.963)	(2.320)	(2.158)	(0.082)	(0.119)
π_{DIV}	Mean	-6.140	-1.720	0.099	1.840	5.704	11.844	16.408
	EW	1.218	1.201	1.280	1.294	1.511	0.293	0.303
		(2.439)	(2.942)	(3.062)	(2.746)	(2.525)	(1.359)	(1.121)
	VW	0.823	0.878	1.045	1.169	1.129	0.305	0.637
		(2.189)	(3.019)	(3.178)	(2.910)	(2.208)	(1.107)	(2.156)

(*continued*)

Table 2. (*continued*)

		1	2	3	4	5	5-1	10-1
η_{SMB}	Mean	-1.562	0.842	2.342	7.682	7.989	9.550	12.618
	EW	1.272	1.178	1.279	1.230	1.576	0.304	0.361
		(2.775)	(2.880)	(3.128)	(2.539)	(2.408)	(0.914)	(0.837)
	VW	1.116	0.898	0.981	0.991	1.004	-0.112	0.120
		(2.760)	(3.051)	(3.099)	(2.367)	(1.895)	(-0.340)	(0.313)
κ_{SMB}	Mean	-3.636	-1.293	-0.130	1.156	5.243	8.878	12.547
	EW	1.524	1.372	1.289	1.123	1.186	-0.338	-0.268
		(2.645)	(3.094)	(3.065)	(2.524)	(2.235)	(-1.151)	(-0.732)
	VW	1.206	1.138	1.196	0.851	0.903	-0.303	-0.552
		(2.427)	(2.998)	(3.669)	(2.349)	(2.197)	(-0.936)	(-1.406)
π_{SMB}	Mean	-1.646	0.161	1.189	2.512	5.001	6.646	8.772
	EW	1.082	1.162	1.290	1.351	1.594	0.512	0.715
		(2.338)	(2.943)	(2.847)	(2.794)	(2.620)	(1.634)	(1.872)
	VW	0.843	1.031	1.126	1.157	1.232	0.390	0.416
		(2.209)	(3.271)	(3.304)	(2.772)	(2.351)	(1.190)	(1.028)
η_{HML}	Mean	-0.711	0.362	1.100	1.743	4.047	4.758	6.692
	EW	0.825	1.040	1.317	1.443	1.910	1.085	1.409
		(2.345)	(2.686)	(2.987)	(2.676)	(2.858)	(2.750)	(3.018)
	VW	0.794	0.916	1.136	1.276	1.413	0.620	0.591
		(2.720)	(3.235)	(3.035)	(2.706)	(2.414)	(1.498)	(1.162)
κ_{HML}	Mean	-38.456	-8.562	4.062	18.289	77.145	115.601	170.565
	EW	1.787	1.378	1.148	1.104	1.098	-0.689	-0.757
		(3.284)	(3.176)	(2.770)	(2.315)	(1.997)	(-2.358)	(-2.147)
	VW	1.420	1.254	0.991	0.915	0.856	-0.563	-0.620
		(3.359)	(3.671)	(2.999)	(2.320)	(1.806)	(-1.566)	(-1.402)
π_{HML}	Mean	-0.444	0.382	0.807	1.286	2.235	2.679	3.591
	EW	0.667	1.035	1.279	1.540	1.973	1.306	1.656
		(1.720)	(2.667)	(2.850)	(2.943)	(3.088)	(3.924)	(4.141)
	VW	0.531	0.766	1.177	1.415	1.494	0.962	1.368
		(1.807)	(2.682)	(3.242)	(3.134)	(2.564)	(2.476)	(2.777)

Table 3. Portfolios sorted on co-moments: Post-1962 period (1962–2012)

This table reports mean of co-moments and mean monthly returns of portfolios sorted on co-moments of sample stocks. Annual co-moments are estimated as the sample moments using daily data of stock returns, market portfolio returns, and state variables. At the beginning of each year, we form quintile and decile portfolios sorted on each of the estimated co-moments, and monthly equal-weighted (EW) and value-weighted (VW) returns are calculated. β is covariance with the market returns, γ is co-skewness with the market returns, and δ is co-kurtosis with the market returns. η is covariance with a state variable, κ is co-skewness with the market returns and a state variable, and π is co-kurtosis with the squared market returns and a state variable. State variables are the dividend yield (*DIV*), size (*SMB*) and value (*HML*) factors of Fama and French (1993), the three-month T-bill rate (*TB*), and the term spread (*TERM*). Each column of "1" to "5" indicates quintile portfolios in ascending order, and the column "5-1" ("10-1") indicates a zero-investment portfolio that is long in the highest moment quintile (decile) and short in the lowest moment quintile (decile). The mean values of co-moments are shown in the units of corresponding moments of the market returns. The numbers in parentheses are *t*-statistics adjusted using Newey–West (1987) standard errors with 12 lags. The sample period is January 1962 to December 2012.

		1	2	3	4	5	5-1	10-1
β	Mean	0.009	0.426	0.724	1.065	1.733	1.723	2.188
	EW	0.659	0.939	1.117	1.329	1.806	1.147	1.602
		(3.145)	(4.323)	(4.677)	(4.744)	(4.259)	(3.749)	(4.116)
	VW	0.753	0.785	0.905	0.789	0.963	0.210	0.492
		(5.495)	(5.805)	(6.021)	(4.078)	(2.884)	(0.677)	(1.280)
γ	Mean	-12.515	-4.626	-1.872	1.491	7.659	20.174	26.985
	EW	1.648	1.295	1.168	0.989	0.707	-0.942	-1.145
		(4.389)	(5.208)	(5.458)	(4.534)	(2.337)	(-4.327)	(-4.254)
	VW	1.083	1.060	0.981	0.837	0.441	-0.643	-0.784
		(3.299)	(5.070)	(5.589)	(5.033)	(1.904)	(-2.754)	(-2.418)
δ	Mean	-0.093	0.415	0.745	1.136	1.943	2.036	2.638
	EW	0.636	0.992	1.154	1.309	1.760	1.125	1.447
		(2.817)	(4.545)	(4.929)	(4.748)	(4.275)	(3.929)	(4.070)
	VW	0.620	0.787	0.877	0.921	1.064	0.444	0.736
		(3.588)	(5.581)	(5.711)	(4.504)	(3.217)	(1.557)	(1.925)
η_{DIV}	Mean	-22.940	-6.431	-0.921	4.564	20.569	43.509	61.465
	EW	1.195	1.241	1.194	1.169	1.055	-0.140	-0.143
		(3.327)	(5.291)	(5.613)	(5.256)	(3.396)	(-1.316)	(-1.022)
	VW	0.644	0.968	0.957	0.843	0.639	-0.005	0.171
		(1.985)	(4.816)	(5.452)	(4.746)	(2.597)	(-0.027)	(0.767)
κ_{DIV}	Mean	-8.933	-2.181	0.320	3.242	9.360	18.293	24.909
	EW	1.150	1.220	1.191	1.222	1.048	-0.102	-0.166
		(3.287)	(5.434)	(5.607)	(5.178)	(3.135)	(-0.581)	(-0.796)
	VW	0.888	0.944	0.930	0.862	0.593	-0.295	-0.230
		(3.221)	(4.945)	(5.556)	(4.542)	(2.303)	(-1.454)	(-0.943)
π_{DIV}	Mean	-10.302	-2.573	-0.304	1.836	8.074	18.376	26.752
	EW	0.975	1.124	1.189	1.228	1.312	0.336	0.388
		(2.982)	(4.863)	(5.574)	(5.199)	(3.925)	(2.867)	(2.555)
	VW	0.447	0.789	0.941	0.868	0.936	0.489	0.619
		(1.675)	(4.190)	(5.679)	(4.753)	(3.504)	(2.829)	(2.593)
η_{SMB}	Mean	-1.426	1.165	2.857	4.860	9.112	10.538	13.588
	EW	0.935	1.101	1.213	1.283	1.340	0.405	0.483
		(3.992)	(4.995)	(5.444)	(4.668)	(3.117)	(1.297)	(1.228)
	VW	0.846	0.978	1.038	0.833	0.554	-0.292	-0.523
		(4.468)	(5.492)	(4.773)	(2.913)	(1.359)	(-0.815)	(-1.162)
κ_{SMB}	Mean	-63.825	-21.006	-1.216	18.242	62.109	125.933	167.751
	EW	1.425	1.297	1.182	1.091	0.820	-0.605	-0.753
		(3.854)	(5.162)	(5.241)	(4.949)	(2.763)	(-2.828)	(-2.745)
	VW	1.129	1.104	0.900	0.738	0.197	-0.933	-1.152
		(4.027)	(5.658)	(5.152)	(3.857)	(0.667)	(-4.195)	(-4.053)

(*continued*)

Table 3. (*continued*)

		1	2	3	4	5	5-1	10-1
π_{SMB}	Mean	-1.630	-0.070	0.711	1.926	4.687	6.317	9.091
	EW	1.109	1.163	1.180	1.197	1.208	0.099	0.062
		(3.972)	(5.161)	(5.163)	(4.524)	(3.177)	(0.384)	(0.200)
	VW	0.851	0.890	0.923	0.882	0.498	-0.353	-0.674
		(4.081)	(5.718)	(5.016)	(3.782)	(1.372)	(-1.098)	(-1.608)
η_{HML}	Mean	-7.885	-2.267	0.047	2.303	7.316	15.201	20.293
	EW	1.477	1.165	1.063	1.056	1.112	-0.365	-0.597
		(3.699)	(4.271)	(4.603)	(4.650)	(3.882)	(-1.102)	(-1.445)
	VW	0.904	0.780	0.850	0.936	0.874	-0.030	-0.201
		(3.165)	(4.175)	(5.010)	(5.247)	(3.765)	(-0.099)	(-0.491)
κ_{HML}	Mean	-7.214	-1.445	0.451	2.409	7.659	14.873	21.970
	EW	0.874	1.065	1.155	1.290	1.441	0.567	0.660
		(2.647)	(4.686)	(5.399)	(5.555)	(4.067)	(2.889)	(2.662)
	VW	0.586	0.834	0.909	1.035	0.948	0.362	0.296
		(2.311)	(4.770)	(5.401)	(5.501)	(3.386)	(1.829)	(0.975)
π_{HML}	Mean	-2.631	-1.113	-0.397	0.346	1.636	4.268	5.758
	EW	1.480	1.186	1.155	1.069	0.976	-0.504	-0.734
		(3.813)	(4.423)	(4.934)	(4.692)	(3.531)	(-1.692)	(-2.024)
	VW	0.949	0.832	0.824	0.804	0.860	-0.090	-0.308
		(3.374)	(4.456)	(5.139)	(4.391)	(3.717)	(-0.313)	(-0.842)
η_{TB}	Mean	-25.865	-7.752	-1.177	5.647	22.131	47.996	67.391
	EW	1.448	1.085	1.091	1.055	1.444	-0.004	-0.045
		(3.146)	(4.157)	(5.081)	(4.649)	(4.005)	(-0.008)	(-0.068)
	VW	0.849	0.824	0.896	0.970	0.887	0.038	-0.109
		(2.252)	(3.664)	(4.709)	(5.544)	(3.141)	(0.097)	(-0.200)
κ_{TB}	Mean	-6.909	-1.793	0.132	2.104	6.759	13.668	19.080
	EW	1.151	1.221	1.151	1.141	1.162	0.010	0.034
		(3.545)	(5.420)	(5.289)	(4.687)	(3.283)	(0.053)	(0.139)
	VW	0.725	0.859	0.877	0.944	0.758	0.034	0.054
		(2.746)	(4.699)	(5.223)	(4.947)	(2.616)	(0.127)	(0.156)
π_{TB}	Mean	-16.888	-4.992	1.182	6.098	19.557	36.446	50.753
	EW	1.113	1.159	1.166	1.210	1.203	0.090	0.076
		(3.070)	(4.922)	(5.361)	(5.173)	(3.515)	(0.328)	(0.217)
	VW	0.749	0.926	0.929	0.873	0.718	-0.031	-0.269
		(2.776)	(4.838)	(5.455)	(4.503)	(2.500)	(-0.129)	(-0.824)
η_{TERM}	Mean	-13.807	-3.360	0.451	4.700	15.395	29.202	41.227
	EW	1.574	1.088	1.044	1.068	1.361	-0.212	-0.284
		(4.164)	(4.652)	(4.840)	(4.226)	(3.102)	(-0.436)	(-0.436)
	VW	0.977	1.033	0.876	0.849	0.700	-0.277	-0.361
		(3.297)	(5.239)	(4.793)	(3.867)	(1.967)	(-0.672)	(-0.620)
κ_{TERM}	Mean	-7.307	-1.705	-0.221	1.296	5.930	13.237	19.502
	EW	1.260	1.158	1.141	1.188	1.106	-0.154	-0.321
		(3.507)	(4.692)	(5.327)	(5.255)	(3.482)	(-0.756)	(-1.255)
	VW	0.797	0.952	0.908	0.819	0.644	-0.152	-0.304
		(2.625)	(4.710)	(5.475)	(4.629)	(2.534)	(-0.560)	(-0.816)
π_{TERM}	Mean	-11.764	-4.099	0.030	3.526	12.977	24.741	34.147
	EW	1.259	1.205	1.139	1.169	1.107	-0.152	-0.115
		(3.692)	(5.146)	(5.294)	(4.910)	(3.032)	(-0.540)	(-0.320)
	VW	0.784	0.882	0.929	0.958	0.726	-0.059	-0.019
		(2.722)	(4.493)	(5.419)	(5.026)	(2.450)	(-0.222)	(-0.054)

Table 4. Cross-sectional regressions: Full sample period (1926–2012)

This table reports estimates of market prices of co-moments using two-pass cross-sectional regressions. Annual co-moments are estimated as the sample moments using daily data of stock returns, market portfolio returns, and state variables for non-overlapping rolling windows. In the cross-section, we regress daily excess returns of stocks on the estimated co-moments, and time-series averages of the second-stage coefficients are reported. β is covariance with the market returns, γ is co-skewness with the market returns, and δ is co-kurtosis with the market returns. η is covariance with a state variable, κ is co-skewness with the market returns and a state variable, and π is co-kurtosis with the squared market returns and a state variable. State variables are the dividend yield (*DIV*), size (*SMB*) and value (*HML*) factors of Fama and French (1993). The numbers in parentheses are *t*-statistics using Fama-MacBath standard errors, and the numbers in brackets are *t*-statistics using asymptotic standard errors corrected for estimation errors in the first stage. The sample period is January 1926 to December 2012.

State variables		Intercept	β	γ	δ	η	κ	π
Static model	1	15.65	5.62					
		(9.04)	(5.07)					
		[5.47]	[4.70]					
	2	14.91	4.49	-603.90				
		(11.74)	(3.74)	(-4.41)				
		[7.53]	[3.31]	[-4.66]				
	3	14.20	4.00	-720.01	-5743.95			
		(15.65)	(2.12)	(-4.30)	(-0.32)			
		[12.74]	[2.03]	[-4.23]	[-0.32]			
DIV	4	14.87	5.69			-0.61		
		(12.89)	(5.27)			(-2.11)		
		[8.90]	[5.17]			[-1.53]		
	5	14.06	4.68	-617.56		-0.52	1.93	
		(15.84)	(4.05)	(-4.48)		(-1.99)	(0.06)	
		[13.52]	[3.96]	[-4.72]		[-1.64]	[0.06]	
	6	13.52	3.82	-720.43	-4544.78	-0.43	-2.53	-4073.30
		(16.53)	(2.00)	(-4.09)	(-0.22)	(-1.53)	(-0.07)	(-1.00)
		[15.55]	[1.95]	[-3.97]	[-0.22]	[-1.49]	[-0.07]	[-1.01]
SMB	7	11.36	4.80			17.06		
		(11.93)	(3.97)			(6.72)		
		[8.36]	[3.85]			[6.08]		
	8	10.27	3.76	-600.08		14.36	-116.70	
		(14.17)	(2.82)	(-3.60)		(5.48)	(-0.26)	
		[12.81]	[2.63]	[-3.59]		[5.13]	[-0.23]	
	9	10.40	4.84	-677.70	-9522.91	13.63	-179.56	113314.40
		(14.85)	(2.34)	(-3.28)	(-0.45)	(4.20)	(-0.36)	(1.65)
		[13.30]	[2.21]	[-2.97]	[-0.40]	[3.93]	[-0.31]	[1.29]
HML	10	15.85	3.01			2.70		
		(9.31)	(2.14)			(0.99)		
		[5.55]	[2.02]			[0.93]		
	11	14.07	1.66	-762.92		1.33	-551.97	
		(15.91)	(1.10)	(-4.02)		(0.47)	(-1.24)	
		[12.04]	[0.98]	[-4.11]		[0.43]	[-1.26]	
	12	13.42	1.70	-905.65	-25122.43	0.93	-476.35	82348.46
		(17.01)	(0.78)	(-4.08)	(-1.06)	(0.28)	(-1.00)	(1.33)
		[14.50]	[0.74]	[-3.92]	[-1.00]	[0.27]	[-0.95]	[1.23]

(*continued*)

Table 4. (*continued*)

State variables		Intercept	β	γ	δ	η	κ	π
SMB + *HML*	13	11.38	3.42			16.61		
		(15.23)	(2.21)			(6.33)		
		[12.97]	[2.08]			[5.63]		
						3.99		
						(1.27)		
						[0.99]		
	14	10.54	1.98	-741.53		13.13	-190.42	
		(15.57)	(1.21)	(-3.47)		(4.93)	(-0.41)	
		[14.18]	[1.15]	[-3.41]		[4.65]	[-0.37]	
						3.66	-757.72	
						(1.26)	(-1.66)	
						[1.14]	[-1.63]	
	15	10.47	3.09	-833.35	-23582.27	12.67	-332.08	103897.10
		(16.14)	(1.31)	(-3.22)	(-0.88)	(3.80)	(-0.66)	(1.46)
		[14.69]	[1.27]	[-2.98]	[-0.79]	[3.59]	[-0.55]	[1.12]
						3.07	-773.28	34997.18
						(0.89)	(-1.57)	(0.52)
						[0.85]	[-1.46]	[0.45]

Table 5. Cross-sectional regressions: Pre-1962 period (1926–1961)

This table reports estimates of market prices of co-moments using two-pass cross-sectional regressions. Annual co-moments are estimated as the sample moments using daily data of stock returns, market portfolio returns, and state variables for non-overlapping rolling windows. In the cross-section, we regress daily excess returns of stocks on the estimated co-moments, and time-series averages of the second-stage coefficients are reported. β is covariance with the market returns, γ is co-skewness with the market returns, and δ is co-kurtosis with the market returns. η is covariance with a state variable, κ is co-skewness with the market returns and a state variable, and π is co-kurtosis with the squared market returns and a state variable. State variables are the dividend yield (*DIV*), size (*SMB*) and value (*HML*) factors by Fama and French (1993). The numbers in parentheses are t-statistics using Fama-MacBath standard errors, and the numbers in brackets are t-statistics using asymptotic standard errors corrected for estimation errors in the first stage. The sample period is January 1926 to December 1961.

State variables		Intercept	β	γ	δ	η	κ	π
Static model	1	17.66	6.68					
		(10.22)	(4.06)					
		[10.01]	[4.20]					
	2	18.49	5.09	-645.53				
		(11.29)	(2.80)	(-2.91)				
		[10.86]	[2.76]	[-3.12]				
	3	18.37	4.42	-738.59	797.40			
		(11.50)	(1.40)	(-2.86)	(0.03)			
		[11.28]	[1.41]	[-2.75]	[0.03]			
DIV	4	17.88	6.66			-0.12		
		(10.81)	(4.05)			(-0.57)		
		[10.58]	[4.17]			[-0.56]		
	5	18.20	5.36	-634.89		-0.13	-12.42	
		(11.45)	(2.95)	(-2.92)		(-0.60)	(-0.39)	
		[11.20]	[2.88]	[-3.11]		[-0.57]	[-0.39]	
	6	17.56	3.68	-709.16	12746.43	-0.26	-5.27	420.44
		(11.44)	(1.17)	(-2.70)	(0.53)	(-0.97)	(-0.15)	(0.10)
		[11.26]	[1.16]	[-2.59]	[0.53]	[-0.92]	[-0.15]	[0.10]
SMB	7	12.65	5.11			19.84		
		(8.39)	(2.71)			(4.46)		
		[7.12]	[2.62]			[3.98]		
	8	12.08	3.81	-727.77		16.99	-812.35	
		(8.60)	(1.83)	(-2.71)		(3.74)	(-0.98)	
		[7.50]	[1.77]	[-2.84]		[3.57]	[-0.95]	
	9	12.35	3.68	-948.22	8747.64	18.24	-989.66	-41279.96
		(9.30)	(1.07)	(-3.03)	(0.30)	(3.18)	(-1.12)	(-0.42)
		[8.36]	[1.04]	[-2.94]	[0.30]	[2.90]	[-1.08]	[-0.43]
HML	10	18.10	1.75			2.33		
		(11.65)	(0.75)			(0.59)		
		[11.13]	[0.72]			[0.55]		
	11	17.70	0.20	-758.87		1.59	387.15	
		(11.94)	(0.08)	(-2.25)		(0.39)	(0.57)	
		[11.18]	[0.08]	[-2.26]		[0.38]	[0.57]	
	12	16.72	1.45	-933.87	-21600.41	0.47	261.76	75934.65
		(11.85)	(0.39)	(-2.48)	(-0.66)	(0.09)	(0.36)	(0.93)
		[11.72]	[0.39]	[-2.30]	[-0.66]	[0.09]	[0.35]	[0.96]

(*continued*)

Table 5. (*continued*)

State variables		Intercept	β	γ	δ	η	κ	π
SMB + *HML*	13	13.65	1.54			17.19		
		(9.42)	(0.61)			(3.83)		
		[8.00]	[0.59]			[3.49]		
						0.38		
						(0.09)		
						[0.09]		
	14	12.71	-0.26	-732.65		13.37	-757.33	
		(9.54)	(-0.10)	(-2.03)		(2.91)	(-0.90)	
		[8.43]	[-0.09]	[-2.00]		[2.76]	[-0.88]	
						0.91	-203.11	
						(0.23)	(-0.29)	
						[0.22]	[-0.29]	
	15	12.64	0.48	-927.26	-8630.58	15.35	-1070.08	-39459.38
		(9.91)	(0.12)	(-2.23)	(-0.23)	(2.63)	(-1.19)	(-0.38)
		[8.79]	[0.12]	[-2.07]	[-0.23]	[2.45]	[-1.14]	[-0.39]
						1.41	-210.02	55882.35
						(0.28)	(-0.28)	(0.66)
						[0.27]	[-0.27]	[0.68]

Table 6. Cross-sectional regressions: Post-1962 period (1962–2012)

This table reports estimates of market prices of co-moments using two-pass cross-sectional regressions. Annual co-moments are estimated as the sample moments using daily data of stock returns, market portfolio returns, and state variables for non-overlapping rolling windows. In the cross-section, we regress daily excess returns of stocks on the estimated co-moments, and time-series averages of the second-stage coefficients are reported. β is covariance with the market returns, γ is co-skewness with the market returns, and δ is co-kurtosis with the market returns. η is covariance with a state variable, κ is co-skewness with the market returns and a state variable, and π is co-kurtosis with the squared market returns and a state variable. State variables are the dividend yield (*DIV*), size (*SMB*) and value (*HML*) factors of Fama and French (1993), the three-month T-bill rate (*TB*), and the term spread (*TERM*). The numbers in parentheses are *t*-statistics using Fama-MacBath standard errors, and the numbers in brackets are *t*-statistics using asymptotic standard errors corrected for estimation errors in the first stage. The sample period is January 1962 to December 2012.

State variables		Intercept	β	γ	δ	η	κ	π
Static model	1	14.23	4.86					
		(5.29)	(3.26)					
		[3.02]	[2.86]					
	2	12.39	4.07	-574.52				
		(6.75)	(2.54)	(-3.32)				
		[3.92]	[2.12]	[-3.45]				
	3	11.25	3.71	-706.89	-10361.37			
		(10.61)	(1.60)	(-3.22)	(-0.41)			
		[7.44]	[1.47]	[-3.21]	[-0.40]			
DIV	4	12.74	5.00			-0.96		
		(8.04)	(3.50)			(-2.04)		
		[4.92]	[3.33]			[-1.44]		
	5	11.13	4.20	-605.33		-0.79	12.05	
		(10.95)	(2.80)	(-3.40)		(-1.90)	(0.25)	
		[8.23]	[2.74]	[-3.56]		[-1.54]	[0.23]	
	6	10.67	3.91	-728.39	-16750.33	-0.54	-0.60	-7245.35
		(12.15)	(1.64)	(-3.08)	(-0.56)	(-1.25)	(-0.01)	(-1.15)
		[10.74]	[1.58]	[-3.01]	[-0.54]	[-1.22]	[-0.01]	[-1.17]
SMB	7	10.45	4.57			15.10		
		(8.51)	(2.91)			(5.07)		
		[5.36]	[2.82]			[4.65]		
	8	9.00	3.72	-509.94		12.51	374.34	
		(12.17)	(2.15)	(-2.41)		(4.01)	(0.73)	
		[11.81]	[1.95]	[-2.31]		[3.69]	[0.62]	
	9	9.03	5.66	-486.74	-22419.77	10.38	392.28	222439.83
		(12.18)	(2.21)	(-1.77)	(-0.76)	(2.76)	(0.69)	(2.36)
		[10.84]	[2.04]	[-1.54]	[-0.64]	[2.65]	[0.52]	[1.67]
HML	10	14.26	3.90			2.95		
		(5.31)	(2.24)			(0.79)		
		[3.01]	[2.06]			[0.75]		
	11	11.50	2.68	-765.78		1.14	-1214.88	
		(10.60)	(1.41)	(-3.50)		(0.30)	(-2.07)	
		[6.97]	[1.18]	[-3.65]		[0.26]	[-2.12]	
	12	11.09	1.88	-885.72	-27608.57	1.25	-997.36	86875.85
		(12.26)	(0.71)	(-3.28)	(-0.83)	(0.28)	(-1.58)	(0.98)
		[9.12]	[0.65]	[-3.28]	[-0.76]	[0.27]	[-1.48]	[0.87]

(*continued*)

Table 6. (*continued*)

State variables		Intercept	β	γ	δ	η	κ	π
SMB + *HML*	13	9.78	4.75			16.20		
		(12.85)	(2.43)			(5.12)		
		[10.99]	[2.24]			[4.45]		
						6.54		
						(1.44)		
						[1.06]		
	14	9.01	3.55	-747.79		12.96	209.75	
		(13.42)	(1.72)	(-2.87)		(4.07)	(0.40)	
		[13.05]	[1.64]	[-2.81]		[3.82]	[0.33]	
						5.60	-1149.21	
						(1.38)	(-1.89)	
						[1.21]	[-1.85]	
	15	8.93	4.93	-767.06	-34136.40	10.77	188.86	205089.91
		(13.89)	(1.71)	(-2.32)	(-0.91)	(2.75)	(0.33)	(2.11)
		[13.34]	[1.64]	[-2.15]	[-0.79]	[2.65]	[0.24]	[1.45]
						4.24	-1170.88	20254.71
						(0.90)	(-1.78)	(0.21)
						[0.85]	[-1.62]	[0.17]
TB	16	11.36	5.07			-8.19		
		(13.08)	(3.60)			(-0.68)		
		[12.24]	[3.43]			[-0.65]		
	17	10.69	3.76	-529.92		-5.65	265.12	
		(13.03)	(2.49)	(-3.06)		(-0.48)	(0.20)	
		[12.23]	[2.37]	[-3.09]		[-0.45]	[0.21]	
	18	10.46	3.05	-607.85	-6859.71	-2.49	-329.11	-155217.82
		(12.94)	(1.33)	(-2.60)	(-0.27)	(-0.19)	(-0.23)	(-1.00)
		[12.07]	[1.28]	[-2.39]	[-0.24]	[-0.17]	[-0.24]	[-1.00]
TERM	19	13.39	5.00			-1.47		
		(8.02)	(3.55)			(-0.18)		
		[3.96]	[3.39]			[-0.11]		
	20	12.01	3.66	-586.34		-3.49	-2293.18	
		(9.21)	(2.33)	(-3.35)		(-0.48)	(-2.50)	
		[4.80]	[1.99]	[-3.28]		[-0.32]	[-2.40]	
	21	11.48	3.03	-589.51	439.66	-6.20	-2037.62	173255.17
		(10.30)	(1.32)	(-2.47)	(0.02)	(-0.89)	(-1.91)	(1.22)
		[5.81]	[1.25]	[-2.31]	[0.02]	[-0.66]	[-1.80]	[1.01]

Table 7. Implied parameters for relative risk aversion and relative prudence

This table presents values of relative risk aversion (RRA) and relative prudence (RPR) parameters implied by estimates of the cross-sectional regressions reported in Tables 4–6. Implied RRA and RPR are obtained from the following asset pricing equation:

$$E_t\left[R_{t+1}-R_{ft}\right]=i_t\cdot\gamma_t\cdot E_t\left[(R_{t+1}-\mu_t)(R^M_{t+1}-\mu^M_t)\right]-\frac{i_t^2}{2}\cdot\gamma_t\cdot\theta_t\cdot E_t\left[(R_{t+1}-\mu_t)(R^M_{t+1}-\mu^M_t)^2\right]+\cdots\ ,$$

where γ_t is RRA, θ_t is RPR, i_t is the ratio of risky investments to the expected payoff of total investments in the next period, defined as $i_t = w\big/\left\{(1-w)R_{ft}+w\mu^M_t\right\}$, and w denotes the proportion of risky investments in total investments, $W_t - C_t$. To calculate i_t for given w, we use mean annual returns of the market portfolio and the risk-free asset during the sample period. RRA and RPR are calculated for each specification and varying values of w from 0.25 to 2. Panel A shows implied parameters from the full-period (1926–2012) estimation result, and Panel B and Panel C show results for the pre- and post-1962 periods, respectively.

Panel A: Full sample period (1926-2012)

State variables		w = 0.25		w = 0.5		w = 0.75		w = 1		w = 1.25		w = 1.5		w = 1.75		w = 2	
		RRA	RPR	RRA	RPR	RRA	RPR	RRA	RPR	RRA	RPR	RRA	RPR	RRA	RPR	RRA	RPR
Static model	1	23.71		12.08		8.21		6.27		5.10		4.33		3.77		3.36	
	2	18.97	1135.36	9.66	578.46	6.56	392.83	5.01	300.01	4.08	244.32	3.46	207.19	3.02	180.68	2.69	160.79
DIV	4	24.01		12.23		8.31		6.35		5.17		4.38		3.82		3.40	
	5	19.76	1114.37	10.07	567.77	6.84	385.57	5.22	294.46	4.25	239.80	3.61	203.36	3.15	177.34	2.80	157.81
SMB	7	20.26		10.32		7.01		5.35		4.36		3.70		3.22		2.87	
	8	15.88	1347.91	8.09	686.75	5.49	466.37	4.20	356.18	3.42	290.06	2.90	245.98	2.53	214.50	2.25	190.89
HML	10	12.70		6.47		4.40		3.36		2.73		2.32		2.02		1.80	
	11	6.99	3890.16	3.56	1982.02	2.42	1345.97	1.85	1027.95	1.51	837.13	1.28	709.93	1.11	619.06	0.99	550.91
SMB + *HML*	13	14.44		7.36		5.00		3.82		3.11		2.64		2.30		2.05	
	14	8.35	3168.26	4.25	1614.21	2.89	1096.20	2.21	837.19	1.80	681.79	1.52	578.18	1.33	504.18	1.18	448.68

(*continued*)

Table 7. (*continued*)

Panel B: Pre-1962 period (1926-1961)																	
State variables		*w* = 0.25		*w* = 0.5		*w* = 0.75		*w* = 1		*w* = 1.25		*w* = 1.5		*w* = 1.75		*w* = 2	
		RRA	RPR	RRA	RPR	RRA	RPR	RRA	RPR	RRA	RPR	RRA	RPR	RRA	RPR	RRA	RPR
Static model	1	27.82		14.29		9.77		7.52		6.17		5.26		4.62		4.13	
	2	21.21	1055.11	10.89	541.79	7.45	370.68	5.73	285.12	4.70	233.79	4.01	199.57	3.52	175.13	3.15	156.79
DIV	4	27.75		14.25		9.75		7.50		6.15		5.25		4.61		4.12	
	5	22.32	986.11	11.46	506.35	7.84	346.44	6.03	266.48	4.95	218.50	4.22	186.52	3.71	163.67	3.32	146.54
SMB	7	21.29		10.93		7.48		5.75		4.72		4.03		3.53		3.16	
	8	15.88	1589.47	8.15	816.17	5.58	558.41	4.29	429.52	3.52	352.19	3.00	300.64	2.63	263.82	2.36	236.20
HML	10	7.29		3.74		2.56		1.97		1.62		1.38		1.21		1.08	
	11	0.84	31239.66	0.43	16041.17	0.30	10975.00	0.23	8441.92	0.19	6922.07	0.16	5908.84	0.14	5185.10	0.13	4642.30
SMB + *HML*	13	6.39		3.28		2.25		1.73		1.42		1.21		1.06		0.95	
Panel C: Post-1962 period (1962-2012)																	
State variables		*w* = 0.25		*w* = 0.5		*w* = 0.75		*w* = 1		*w* = 1.25		*w* = 1.5		*w* = 1.75		*w* = 2	
		RRA	RPR	RRA	RPR	RRA	RPR	RRA	RPR	RRA	RPR	RRA	RPR	RRA	RPR	RRA	RPR
Static model	1	20.74		10.51		7.10		5.39		4.37		3.69		3.20		2.84	
	2	17.34	1204.87	8.79	610.56	5.94	412.46	4.51	313.41	3.66	253.98	3.09	214.36	2.68	186.05	2.37	164.83
DIV	4	21.31		10.80		7.29		5.54		4.49		3.79		3.29		2.92	
	5	17.91	1229.36	9.08	622.97	6.13	420.84	4.66	319.78	3.78	259.14	3.19	218.71	2.77	189.84	2.45	168.18
SMB	7	19.51		9.88		6.68		5.07		4.11		3.47		3.01		2.67	
	8	15.87	1168.37	8.04	592.07	5.43	399.96	4.13	303.91	3.35	246.28	2.82	207.86	2.45	180.42	2.17	159.84
HML	10	16.62		8.42		5.69		4.32		3.50		2.96		2.57		2.27	
	11	11.44	2434.63	5.80	1233.74	3.92	833.44	2.98	633.29	2.41	513.20	2.04	433.14	1.77	375.95	1.57	333.06
SMB + *HML*	13	20.26		10.27		6.94		5.27		4.27		3.60		3.13		2.77	
	14	15.15	1795.03	7.68	909.62	5.19	614.48	3.94	466.92	3.19	378.38	2.70	319.35	2.34	277.19	2.07	245.56
TB	16	21.61		10.95		7.40		5.62		4.56		3.85		3.34		2.96	
	17	16.01	1203.52	8.12	609.88	5.48	412.00	4.17	313.06	3.38	253.69	2.85	214.12	2.47	185.85	2.19	164.64
TERM	19	21.33		10.81		7.30		5.55		4.50		3.79		3.29		2.92	
	20	15.59	1367.72	7.90	693.08	5.34	468.21	4.06	355.77	3.29	288.30	2.77	243.33	2.41	211.20	2.13	187.11

Table 8. Co-moments and returns of portfolios sorted on firm-size

This table reports estimated co-moments and mean monthly returns of portfolios sorted on firm size. Daily and monthly value-weighted returns on quintile and decile portfolios based on firm size are obtained from the Kenneth French's website. Co-moments of each portfolio are estimated as the sample moments using daily portfolio returns, market portfolio returns, and state variables. β is covariance with the market returns, γ is co-skewness with the market returns, and δ is co-kurtosis with the market returns. η is covariance with a state variable, and κ is co-skewness with the market returns and a state variable. State variables are the dividend yield (*DIV*), size (*SMB*) and value (*HML*) factors of Fama and French (1993), the three-month T-bill rate (*TB*), and the term spread (*TERM*). Each column of "1" to "5" indicates quintile portfolios in ascending order, and the column "5-1" ("10-1") indicates a zero-investment portfolio that is long in the largest size quintile (decile) and short in the smallest size quintile (decile). The values of co-moments are shown in the units of corresponding moments of the market returns. The numbers in parentheses are *t*-statistics adjusted using Newey–West (1987) standard errors. Panel A shows results for the period July 1926 to December 1961, and Panel B shows results for the period January 1962 to December 2012.

Panel A: Pre-1962 period (1926.07-1961.12)							
	1	2	3	4	5	5-1	10-1
Return	1.575	1.414	1.309	1.158	0.977	-0.598	-0.888
	(2.432)	(2.701)	(2.842)	(2.807)	(2.951)	(-1.485)	(-1.751)
β	0.975	1.004	1.028	1.043	0.986	0.011	0.068
	(13.138)	(12.956)	(13.532)	(12.973)	(13.037)	(0.277)	(1.466)
γ	-0.027	0.863	0.470	0.898	1.115	1.142	1.165
	(-0.012)	(0.340)	(0.217)	(0.390)	(0.629)	(1.140)	(0.953)
δ	0.941	1.058	1.024	1.083	0.978	0.038	0.089
	(3.758)	(3.368)	(3.772)	(3.581)	(3.719)	(0.390)	(0.819)
η_{DIV}	0.029	0.824	0.931	1.309	0.950	0.921	1.124
	(0.032)	(0.676)	(0.928)	(1.205)	(1.120)	(2.340)	(2.323)
η_{SMB}	1.836	1.157	0.430	-0.291	-1.195	-3.031	-3.305
	(6.176)	(5.172)	(2.107)	(-1.413)	(-5.510)	(-10.482)	(-10.050)
κ_{SMB}	0.195	-0.227	-0.493	-0.740	-1.045	-1.240	-1.580
	(0.259)	(-0.385)	(-0.966)	(-1.464)	(-2.139)	(-2.179)	(-2.214)
η_{HML}	1.423	1.314	1.241	1.211	0.950	-0.472	-0.466
	(10.997)	(10.817)	(10.958)	(10.757)	(10.332)	(-6.882)	(-5.689)

Panel B: Post-1962 period (1962.01-2012.12)							
	1	2	3	4	5	5-1	10-1
Return	1.107	1.109	1.076	1.019	0.821	-0.287	-0.339
	(3.958)	(4.737)	(4.952)	(4.922)	(4.354)	(-1.349)	(-1.383)
β	0.811	0.945	0.941	0.969	1.011	0.200	0.324
	(14.605)	(15.624)	(15.711)	(15.150)	(15.278)	(8.657)	(11.878)
γ	-1.055	-0.994	-0.966	-0.997	-1.014	0.041	0.078
	(-2.048)	(-1.776)	(-1.618)	(-1.445)	(-1.169)	(0.107)	(0.180)
δ	0.749	0.836	0.859	0.944	1.043	0.294	0.388
	(2.635)	(2.660)	(2.557)	(2.430)	(2.205)	(1.502)	(1.761)
η_{DIV}	-26.716	-13.818	-9.410	-6.278	4.620	31.337	38.659
	(-2.402)	(-1.147)	(-0.841)	(-0.553)	(0.389)	(4.410)	(4.728)
η_{SMB}	2.518	2.206	1.275	0.187	-1.910	-4.428	-4.531
	(12.872)	(11.705)	(7.517)	(1.016)	(-6.946)	(-14.129)	(-13.092)
η_{HML}	-0.515	-0.772	-0.875	-0.863	-1.052	-0.537	-0.657
	(-3.913)	(-5.418)	(-6.346)	(-6.419)	(-8.348)	(-9.869)	(-10.473)
κ_{HML}	0.032	-0.484	-0.693	-0.893	-1.094	-1.125	-1.347
	(0.028)	(-0.426)	(-0.624)	(-0.767)	(-0.891)	(-2.819)	(-2.862)
η_{TB}	-1.563	-1.330	-1.292	-1.141	-0.861	0.702	0.859
	(-2.472)	(-2.083)	(-2.133)	(-1.941)	(-1.654)	(2.165)	(2.414)
κ_{TB}	-1.246	-1.391	-1.228	-1.144	-0.883	0.363	0.106
	(-5.218)	(-5.421)	(-5.052)	(-4.575)	(-3.701)	(5.278)	(1.319)
η_{TERM}	1.491	1.358	1.330	1.194	0.883	-0.608	-0.705
	(3.137)	(2.893)	(3.015)	(2.832)	(2.441)	(-2.223)	(-2.329)
κ_{TERM}	1.002	1.184	1.098	1.111	0.979	-0.023	0.252
	(3.759)	(4.065)	(3.854)	(3.692)	(3.185)	(-0.210)	(1.929)

Table 9. Co-moments and returns of portfolios sorted on book-to-market ratio

This table reports estimated co-moments and mean monthly returns of portfolios sorted on book-to-market ratio. Daily and monthly value-weighted returns on quintile and decile portfolios based on book-to-market ratio are obtained from the Kenneth French's website. Co-moments of each portfolio are estimated as the sample moments using daily portfolio returns, market portfolio returns, and state variables. β is covariance with the market returns, γ is co-skewness with the market returns, and δ is co-kurtosis with the market returns. η is covariance with a state variable, and κ is co-skewness with the market returns and a state variable. State variables are the dividend yield (*DIV*), size (*SMB*) and value (*HML*) factors of Fama and French (1993), the three-month T-bill rate (*TB*), and the term spread (*TERM*). Each column of "1" to "5" indicates quintile portfolios in ascending order, and the column "5-1" ("10-1") indicates a zero-investment portfolio that is long in the highest book-to-market quintile (decile) and short in the lowest book-to-market quintile (decile). The values of co-moments are shown in the units of corresponding moments of the market returns. The numbers in parentheses are *t*-statistics adjusted using Newey–West (1987) standard errors. Panel A shows results for the period July 1926 to December 1961, and Panel B shows results for the period January 1962 to December 2012.

Panel A: Pre-1962 period (1926.07-1961.12)							
	1	2	3	4	5	5-1	10-1
Return	0.973	0.929	1.109	1.190	1.394	0.421	0.517
	(2.941)	(2.728)	(2.716)	(2.544)	(2.578)	(1.337)	(1.197)
β	0.997	0.947	1.029	1.221	1.372	0.375	0.512
	(12.590)	(12.658)	(12.372)	(13.521)	(14.926)	(10.205)	(10.291)
γ	0.639	0.810	2.049	2.616	1.593	0.954	0.869
	(0.345)	(0.473)	(0.904)	(1.051)	(0.642)	(1.001)	(0.652)
δ	0.999	0.952	1.111	1.208	1.238	0.239	0.373
	(3.534)	(3.772)	(3.592)	(3.836)	(3.916)	(1.949)	(2.251)
η_{SMB}	-1.092	-0.956	-1.146	-1.197	-0.585	0.507	1.048
	(-5.203)	(-4.551)	(-4.800)	(-4.297)	(-2.358)	(4.076)	(5.205)
η_{HML}	0.830	0.896	1.180	1.652	2.117	1.287	1.617
	(9.411)	(9.893)	(10.145)	(11.465)	(13.316)	(15.616)	(15.444)
Panel B: Post-1962 period (1962.01-2012.12)							
	1	2	3	4	5	5-1	10-1
Return	0.804	0.899	0.931	1.062	1.217	0.413	0.539
	(3.813)	(4.757)	(5.098)	(5.596)	(5.985)	(2.517)	(2.469)
β	1.054	0.954	0.928	0.898	0.944	-0.110	-0.095
	(16.436)	(14.952)	(14.223)	(13.377)	(14.226)	(-5.322)	(-3.624)
γ	-0.891	-1.058	-0.962	-1.115	-1.153	-0.262	-0.315
	(-1.158)	(-1.255)	(-1.265)	(-1.382)	(-1.460)	(-2.115)	(-1.914)
δ	0.992	0.995	0.976	0.998	0.980	-0.012	-0.022
	(2.305)	(2.183)	(2.335)	(2.251)	(2.316)	(-0.409)	(-0.554)
η_{SMB}	-1.341	-1.228	-1.028	-1.008	-0.644	0.697	1.113
	(-6.125)	(-5.403)	(-5.312)	(-3.917)	(-2.771)	(8.686)	(10.961)
κ_{SMB}	0.365	1.912	1.091	1.983	2.607	2.241	2.818
	(0.052)	(0.249)	(0.157)	(0.268)	(0.367)	(3.042)	(3.123)
η_{HML}	-1.661	-0.870	-0.437	0.012	0.203	1.865	2.120
	(-12.392)	(-7.891)	(-3.501)	(0.089)	(1.450)	(19.654)	(18.684)
κ_{TB}	-0.714	-0.880	-1.152	-1.273	-1.441	-0.726	-0.885
	(-3.250)	(-3.745)	(-4.466)	(-4.617)	(-5.221)	(-8.629)	(-7.902)
κ_{TERM}	0.758	1.024	1.224	1.404	1.534	0.776	0.995
	(2.577)	(3.389)	(4.026)	(4.429)	(4.834)	(8.117)	(7.868)

Table 10. Co-moments and returns of momentum portfolios

This table reports estimated co-moments and mean monthly returns of momentum portfolios. At the beginning of each month *t*, we form quintile and decile portfolios sorted on prior returns in months *t*-7 to *t*-2, then monthly and daily value-weighted returns are calculated. Co-moments of each portfolio are estimated as the sample moments using daily portfolio returns, market portfolio returns, and state variables. β is covariance with the market returns, γ is co-skewness with the market returns, and δ is co-kurtosis with the market returns. η is covariance with a state variable, and κ is co-skewness with the market returns and a state variable. State variables are the dividend yield (*DIV*), size (*SMB*) and value (*HML*) factors of Fama and French (1993), the three-month T-bill rate (*TB*), and the term spread (*TERM*). Each column of “1” to “5” indicates quintile portfolios in ascending order, and the column “5-1” (“10-1”) indicates zero-investment portfolio that is long in the highest prior return quintile (decile) and short in the lowest prior return quintile (decile). The values of co-moments are shown in the units of corresponding moments of the market returns. The numbers in parentheses are *t*-statistics adjusted using Newey–West (1987) standard errors. Panel A shows results of the period August 1926 to December 1961, and Panel B shows results of the period January 1962 to December 2012.

Panel A: Pre-1962 period (1926.08-1961.12)							
	1	2	3	4	5	5-1	10-1
Return	0.887	0.997	1.006	1.097	1.299	0.412	0.517
	(1.740)	(2.120)	(2.591)	(3.321)	(3.433)	(0.990)	(0.900)
β	1.241	1.118	1.052	1.009	1.080	-0.161	-0.166
	(12.669)	(12.463)	(13.144)	(13.696)	(12.727)	(-2.982)	(-2.367)
γ	3.136	2.325	1.839	0.635	0.039	-3.097	-3.766
	(1.251)	(1.111)	(0.894)	(0.377)	(0.020)	(-2.836)	(-2.617)
δ	1.275	1.159	1.064	0.945	1.031	-0.245	-0.304
	(3.867)	(3.872)	(3.831)	(4.029)	(3.549)	(-2.505)	(-2.245)
κ_{DIV}	1.451	1.145	1.140	0.880	0.996	-0.455	-0.502
	(1.390)	(1.390)	(1.294)	(1.343)	(1.209)	(-1.876)	(-1.776)
κ_{HML}	1.707	1.492	1.182	0.766	0.716	-0.991	-1.196
	(2.380)	(2.456)	(2.003)	(1.711)	(1.380)	(-2.692)	(-2.306)
Panel B: Post-1962 period (1962.01-2012.12)							
	1	2	3	4	5	5-1	10-1
Return	0.414	0.831	0.892	0.899	1.079	0.665	1.449
	(1.319)	(3.694)	(4.904)	(4.921)	(4.631)	(2.724)	(4.114)
β	1.247	1.049	0.959	0.953	1.096	-0.151	-0.164
	(14.454)	(15.165)	(14.859)	(15.060)	(16.463)	(-3.221)	(-2.997)
γ	-0.877	-0.778	-0.821	-1.032	-1.242	-0.365	-0.262
	(-1.254)	(-1.190)	(-1.111)	(-1.227)	(-1.453)	(-1.331)	(-0.748)
δ	1.141	0.980	0.975	0.990	1.031	-0.110	-0.129
	(2.887)	(2.680)	(2.388)	(2.164)	(2.192)	(-0.857)	(-0.962)
η_{HML}	-1.045	-0.858	-0.782	-0.850	-1.437	-0.392	-0.691
	(-4.215)	(-4.909)	(-6.087)	(-7.642)	(-9.303)	(-1.977)	(-2.749)
η_{TB}	-1.879	-1.203	-1.071	-0.970	-0.868	1.012	1.457
	(-2.297)	(-1.987)	(-1.976)	(-1.763)	(-1.336)	(1.581)	(1.806)
κ_{TB}	-1.815	-1.324	-1.053	-0.839	-0.781	1.035	1.240
	(-4.817)	(-4.486)	(-4.178)	(-3.664)	(-3.537)	(4.933)	(5.138)
η_{TERM}	1.691	1.328	1.086	0.888	0.631	-1.060	-1.392
	(3.020)	(3.225)	(2.873)	(2.251)	(1.299)	(-2.350)	(-2.435)
κ_{TERM}	1.704	1.266	1.092	0.911	0.675	-1.029	-1.247
	(4.278)	(3.891)	(3.641)	(3.057)	(2.153)	(-4.690)	(-4.986)

Table 11. Co-moments and returns of portfolios sorted on idiosyncratic volatility

This table reports estimated co-moments and mean monthly returns of portfolios sorted on idiosyncratic volatility. Following Ang et al. (2006), we define idiosyncratic volatility of a stock for month *t* as the standard deviation of residuals from regression of daily excess returns during the month *t* on the Fama–French three factors. At the beginning of each month *t*, we form quintile and decile portfolios sorted on idiosyncratic volatility in month *t*-1, then monthly and daily value-weighted returns are calculated. Co-moments of each portfolio are estimated as the sample moments using daily portfolio returns, market portfolio returns, and state variables. β is covariance with the market returns, γ is co-skewness with the market returns, and δ is co-kurtosis with the market returns. η is covariance with a state variable, and κ is co-skewness with the market returns and a state variable. State variables are the dividend yield (*DIV*), size (*SMB*) and value (*HML*) factors of Fama and French (1993), the three-month T-bill rate (*TB*), and the term spread (*TERM*). Each column of "1" to "5" indicates quintile portfolios in ascending order, and the column "5-1" ("10-1") indicates a zero-investment portfolio that is long in the highest idiosyncratic volatility quintile (decile) and short in the lowest idiosyncratic volatility quintile (decile). The values of co-moments are shown in the units of corresponding moments of the market returns. The numbers in parentheses are *t*-statistics adjusted using Newey–West (1987) standard errors. Panel A shows results of the period August 1926 to December 1961, and Panel B shows results of the period January 1962 to December 2012.

Panel A: Pre-1962 period (1926.08-1961.12)							
	1	2	3	4	5	5-1	10-1
Return	1.038	1.075	1.063	1.046	0.901	-0.137	-0.120
	(3.276)	(2.691)	(2.382)	(2.184)	(1.849)	(-0.480)	(-0.320)
β	0.919	1.135	1.194	1.122	1.024	0.105	0.089
	(13.116)	(12.791)	(12.832)	(12.522)	(10.930)	(1.745)	(1.025)
γ	1.094	1.241	1.513	1.329	0.221	-0.873	-0.495
	(0.654)	(0.569)	(0.615)	(0.514)	(0.105)	(-0.723)	(-0.264)
δ	0.909	1.158	1.246	1.203	0.955	0.045	-0.091
	(3.768)	(3.886)	(3.529)	(3.222)	(3.359)	(0.463)	(-0.567)
η_{SMB}	-1.175	-1.016	-0.559	0.285	0.691	1.866	1.998
	(-5.650)	(-4.162)	(-2.352)	(1.244)	(1.979)	(5.848)	(4.110)
η_{HML}	0.875	1.229	1.327	1.307	1.196	0.322	0.389
	(10.224)	(10.332)	(10.623)	(10.679)	(9.493)	(3.740)	(2.975)
Panel B: Post-1962 period (1962.01-2012.12)							
	1	2	3	4	5	5-1	10-1
Return	0.875	0.927	0.997	0.678	0.011	-0.864	-1.244
	(5.232)	(4.487)	(3.973)	(2.109)	(0.029)	(-2.621)	(-3.261)
β	0.866	1.068	1.241	1.394	1.372	0.506	0.484
	(14.752)	(15.405)	(15.572)	(15.280)	(14.587)	(10.480)	(9.720)
γ	-0.937	-1.039	-1.152	-1.071	-1.144	-0.207	-0.518
	(-1.177)	(-1.345)	(-1.350)	(-1.347)	(-1.529)	(-0.853)	(-2.280)
δ	0.928	1.034	1.172	1.261	1.249	0.321	0.357
	(2.146)	(2.435)	(2.470)	(2.738)	(2.915)	(2.856)	(3.840)
η_{DIV}	2.417	2.199	-3.023	-2.513	-32.631	-35.047	-44.188
	(0.228)	(0.187)	(-0.216)	(-0.141)	(-1.840)	(-3.096)	(-3.303)
κ_{DIV}	-0.406	-1.596	-2.659	-2.933	-3.439	-3.033	-3.217
	(-0.207)	(-0.834)	(-1.192)	(-1.043)	(-1.273)	(-2.619)	(-2.520)
η_{SMB}	-1.722	-0.854	0.106	1.264	2.148	3.870	4.145
	(-6.962)	(-4.206)	(0.459)	(4.707)	(7.469)	(12.620)	(12.826)
η_{HML}	-0.717	-0.961	-1.402	-1.712	-1.634	-0.916	-0.849
	(-7.437)	(-6.439)	(-6.941)	(-6.217)	(-5.681)	(-4.214)	(-3.726)
η_{TB}	-0.792	-0.841	-1.206	-1.668	-2.700	-1.907	-2.642
	(-1.762)	(-1.453)	(-1.719)	(-1.974)	(-2.885)	(-3.000)	(-3.545)
κ_{TB}	-0.888	-1.260	-1.446	-1.694	-1.753	-0.866	-0.769
	(-4.235)	(-4.577)	(-4.649)	(-4.637)	(-4.441)	(-4.159)	(-3.382)
η_{TERM}	0.856	0.934	1.181	1.407	1.930	1.075	1.247
	(2.644)	(2.321)	(2.357)	(2.314)	(2.786)	(2.145)	(2.094)

Table 12. Co-moments and returns of portfolios sorted on failure probability

This table reports estimated co-moments and mean monthly returns of portfolios sorted on failure probability. We construct a monthly measure of failure probability of each stock following the model in Campbell et al. (2008), using quarterly Compustat data. At the beginning of each month *t*, we form quintile and decile portfolios sorted on the failure probability in month *t*-1, then monthly and daily value-weighted returns are calculated. Co-moments of each portfolio are estimated as the sample moments using daily portfolio returns, market portfolio returns, and state variables. β is covariance with the market returns, γ is co-skewness with the market returns, and δ is co-kurtosis with the market returns. η is covariance with a state variable, and κ is co-skewness with the market returns and a state variable. State variables are the dividend yield (*DIV*), size (*SMB*) and value (*HML*) factors of Fama and French (1993), the three-month T-bill rate (*TB*), and the term spread (*TERM*). Each column of "1" to "5" indicates quintile portfolios in ascending order, and the column "5-1" ("10-1") indicates a zero-investment portfolio that is long in the highest failure probability quintile (decile) and short in the lowest failure probability quintile (decile). The values of co-moments are shown in the units of corresponding moments of the market returns. The numbers in parentheses are *t*-statistics adjusted using Newey–West (1987) standard errors. The results cover the period September 1972 to December 2012.

	Post-1972 period (1972.09-2012.12)						
	1	2	3	4	5	5-1	10-1
Return	0.994	0.993	0.954	0.953	0.319	-0.675	-0.776
	(5.019)	(4.479)	(3.664)	(2.884)	(0.673)	(-1.772)	(-1.868)
β	0.916	0.988	1.122	1.294	1.413	0.497	0.460
	(14.094)	(14.131)	(13.893)	(13.611)	(13.433)	(7.758)	(6.756)
γ	-1.075	-0.957	-0.937	-0.920	-0.949	0.126	0.400
	(-1.239)	(-1.210)	(-1.279)	(-1.187)	(-1.476)	(0.284)	(0.566)
δ	0.958	0.994	1.072	1.199	1.187	0.228	0.113
	(2.004)	(2.241)	(2.532)	(2.717)	(3.287)	(1.173)	(0.357)
η_{DIV}	2.045	0.229	0.532	1.905	-0.628	-2.672	-6.178
	(1.267)	(0.138)	(0.271)	(0.828)	(-0.222)	(-1.263)	(-2.327)
κ_{DIV}	-0.680	-0.874	-1.235	-1.657	-1.963	-1.282	-1.421
	(-1.842)	(-2.299)	(-2.820)	(-3.186)	(-3.151)	(-3.318)	(-3.406)
η_{SMB}	-1.321	-1.208	-0.893	-0.394	0.817	2.138	2.405
	(-5.866)	(-6.325)	(-4.803)	(-1.942)	(3.553)	(7.336)	(6.653)
η_{TB}	-0.821	-0.936	-1.085	-1.127	-1.832	-1.011	-1.534
	(-1.380)	(-1.449)	(-1.474)	(-1.283)	(-1.659)	(-1.260)	(-1.690)
κ_{TB}	-0.677	-0.939	-1.415	-1.870	-2.063	-1.385	-1.292
	(-3.780)	(-4.329)	(-5.070)	(-5.463)	(-5.251)	(-5.623)	(-5.091)
η_{TERM}	0.750	0.861	1.217	1.390	1.866	1.116	1.140
	(1.720)	(1.828)	(2.239)	(2.182)	(2.380)	(1.928)	(1.704)
κ_{TERM}	0.743	0.955	1.417	1.807	1.683	0.940	0.613
	(1.998)	(2.416)	(3.052)	(3.344)	(2.817)	(2.587)	(1.599)